Math 31: Abstract Algebra
Fall 2017 - Homework 1

Return date: Wednesday 09/20/17

keywords: *operations - examples and properties*

Instructions: Write your answers neatly and clearly on straight-edged paper, use complete sentences and label any diagrams. Please show your work; no credit is given for solutions without work or justification.

exercise 1. *(6 points)* Which of the following rules are operations on the indicated set? Justify your answer in each case.

a) $a * b = a \cdot b$, on the set $\{n \in \mathbb{Z} : n \leq 0\}$.

b) $(a, c) * (b, d) = (a + b, c + d)$, on the set $\mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}$.

c) $a * b = |a^2 - b|$, on the set $\{n \in \mathbb{Z} : n \geq 0\}$.

exercise 2. *(6 points)* Each of the following is an operation $*$ on $\mathbb{R}$.

a) $x * y = |x + y|$.

b) $x * y = x \cdot (y + 1)$.

c) $x * y = \min\{x + 1, y + 1\}$.

Explain whether or not

i) the operation is commutative,

ii) the operation is associative,

iii) $\mathbb{R}$ has an identity element with respect to $*$,

iv) every $x \in \mathbb{R}$ has an inverse with respect to $*$.

exercise 3. *(8 points)* Let A be the two-element set $A = \{a, b\}$. Write a table of all 16 possible operations on A using the format explained in the book on page 20. Label these operations Op_1 to Op_{16}. Then

a) among these operations, identify those that are commutative,

b) identify the operations that have an identity element,

c) identify the operations for which each element has an inverse.

Note: Example solutions to similar problems can be found at the end of Chapter 2 of the textbook.