Name: ______________________________

Math 31 Midterm

October 30, 2009

Instructions: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own.

____
Signature

Question	Points	Score
1	14	
2	16	
3	18	
4	12	
5	15	
Total:	75	

1. Mark each of the following statements TRUE or FALSE. You do not need to justify your answers.

   (a) [2 points] Abelian groups are cyclic.

   (b) [2 points] If $K \leq H$ and $H \leq G$, then $K \leq G$.

   (c) [2 points] If G is a cyclic group and $K \leq G$, then K is a normal subgroup.

   (d) [2 points] If $\sigma \in \mathcal{S}_n$ is an odd permutation, then so is σ^{-1}.

   (e) [2 points] If $|G| = n$, $|H| = m$, then $|G \oplus H| = n + m$.

   (f) [2 points] If G is a group and $a \in G$, then $Z(G) \leq C(a)$.

   (g) [2 points] If $H \trianglelefteq G$, then $ghg^{-1} = h$ for all $h \in H$ and $g \in G$.

2. Give a complete definition of each of the following terms.
   (a) [5 points] Cyclic group. (You do not need to define a group, just what makes a group cyclic.)

   (b) [5 points] The center of a group. (Again, you do not need to define a group.)

   (c) [6 points] An automorphism. (If you refer to any other kind of map, you'll need to define it.)

3. Examples.

   (a) [4 points] Give an example of an infinite group that is not abelian.

   (b) [6 points] Give an example of a non-trivial group homomorphism that is not an isomorphism.

   (c) [8 points] Give an example of a group, G and subgroups H and K of G so that K is normal in G, but H is not. (So, $H \leq G$, $H \ntrianglelefteq G$ and $K \trianglelefteq G$.)

4. [12 points] Let $G = Z_{20}$. Clearly list all of the subgroups of G and determine its subgroup lattice. (You may either list a subgroup by specifying all of its elements, or, if the subgroup is cyclic, you may give one of its generators and the order of the subgroup).

5. Consider the direct product $G = Z_{12} \oplus Z_{35}$. Note: Your explanations below should reference the main idea of some theorem. You do not need to give a complete proof, or even state the theorem completely, as long as you give me some idea of how you know the answers.

   (a) [5 points] Is G cyclic? Give a brief explanation.

   (b) [5 points] Is $G \cong Z_{60} \oplus Z_7$? Give a brief explanation.

   (c) [5 points] Is $G \cong Z_6 \oplus Z_{70}$? Give a brief explanation.