Instructions: You are encouraged to work out solutions to these problems in groups! Discuss the problems with your classmates and/or your instructor. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper (this part should be done on your own) and write down the names of the classmates with whom you worked. Be sure to use *complete sentences*. Note: Proofs should contain *words*, not just symbols.

1. (Chapter 18, Exercise 30) Show that $3x^2 + 4x + 3 \in Z_5[x]$ factors as $(3x + 2)(x + 4)$ and $(4x + 1)(2x + 3)$. Explain why this does not contradict the corollary of Theorem 18.3.

2. (a) (Chapter 19, Exercise 2) (Subspace Test) Prove that a nonempty subset U of a vector space V over a field F is a subspace of V if, for every u and u' in U and every a in F, $u + u' \in U$ and $au \in U$.

   (b) (Chapter 19, Exercise 28) Let V and W be vector spaces over a field F and let T be a linear transformation from V to W ($T : V \to W$). Prove that the image of V under T is a subspace of W.

3. (Chapter 20, Exercise 2) Show $\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2} + \sqrt{3})$.

4. (a) (Chapter 19, Exercise 22) If V is a vector space of dimension n over the field Z_p, how many elements are in V? (Give a brief explanation, but no "proof" is required here.)

   (b) (from Chapter 20, Exercise 8) Let $F = Z_2$ and let $f(x) = x^3 + x + 1 \in F[x]$. Suppose that α is a zero of $f(x)$ in some extension of F. The field $F(\alpha)$ can be viewed as a vector space over F. What is the dimension of this vector space? Give a basis for $F(\alpha)$ over F.

   (c) How many elements does $F(\alpha)$ have? Express each member of $F(\alpha)$ in terms of α.

   (d) Write out a complete multiplication table for $F(\alpha)$.