Instructions: You are encouraged to work out solutions to these problems in groups! Discuss the problems with your classmates and/or your instructor. After doing so, please write up your solutions legibly on a separate sheet (or sheets) of paper (this part should be done on your own) and write down the names of the classmates with whom you worked. Be sure to use *complete sentences*. Note: Proofs should contain *words*, not just symbols.

1. (Chapter 16, Exercise 12) Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1$ in $Z_7[x]$. Determine the quotient $(q(x))$ and remainder $(r(x))$ upon dividing $f(x)$ by $g(x)$.

2. (Chapter 16, Exercise 40) Let R be a c-ring with unity. If I is a prime ideal of R, prove that $I[x]$ is a prime ideal of $R[x]$.

   Hint: This problem can be done in a number of ways (as usual). One way makes use of homomorphisms. If you choose this method, you may use without proof the fact that if $\varphi : R \to S$ is a ring homomorphism, so is $\Phi : R[x] \to S[x]$ given by $\Phi(a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0) = \varphi(a_n)x^n + \varphi(a_{n-1})x^{n-1} + \cdots + \varphi(a_1)x + \varphi(a_0)$.

3. (Chapter 16, Exercise 42) Prove that $\mathbb{Q}[x]/\langle x^2 - 2\rangle$ is ring-isomorphic to $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$.

   Hint: You can use the fact that evaluation is a homomorphism. That is, if D and R are integral domains, and D is a subring of R, then the map $E_c : D[x] \to R$, for some integral domain R via $E_c(f(x)) = E_c(a_n x^n + \cdots a_0) = f(c) = a_n \cdot c^n + \cdots a_0 \cdot c \in R$ (for $c \in R$) is a homomorphism.

4. (a) (Chapter 17, Exercise 6) Suppose $f(x) \in Z_p[x]$ and is irreducible over Z_p, where p is a prime. If $deg\ f(x) = n$, prove that $Z_p[x]/\langle f(x)\rangle$ is a field with p^n elements.

   (b) (Chapter 17, Exercise 8) Construct a field of order 27.

5. (Chapter 18, Exercise 2) In an integral domain, show a and b are associates if and only if $\langle a\rangle = \langle b\rangle$.

6. (Chapter 18, Exercise 4) In an integral domain, show that the product of an irreducible and a unit is an irreducible.