H.W. 5

Due Monday Feb. 16th

Monday: Trigonometric Integrals

1)

Solve the following indefinite integrals using the techniques shown in Monday's lecture (Section 7.2 in Stewart).

(i)

$$\int \sin^4(\theta)\cos^2(\theta)\,d\theta$$

(ii)

$$\int \tan^3(2\theta)\sec^5(2\theta)\,d\theta$$

(iii)

$$\int \cot^2(\pi\theta)\csc^4(\pi\theta)\,d\theta$$

2)

We've seen general strategies for evaluating certain trigonometric integrals. However, several cases have been omitted in solving the general problem of finding $\int \sin^a(x)\cos^b(x)\,dx$ where a and b might be any integer. This section extends the cases we've covered in class, but uses many of the same techniques.

Where I ask you to describe a process, the style of exposition and level of detail given in the text on pages 473 and 474 is sufficient. However, any description that you feel is sufficiently thorough and clear is likely to receive full credit (if your process works, of course).

A helpful tip for determining a general process: try out a few easy cases on your own until you see a general pattern.

(i) Describe a process to evaluate $\int \big[\tan(x)\big]^{2k}\,dx$ where $k \geq 1$ is any integer (i.e. isolated even powers of tangent).

(ii) Evaluate the following indefinite integral. (Hint: We already know the integral of $\sec(x)$ from our trigonometric integral table. Use integration by parts here with $u = \sec(x)$. Then transform the integral term you get (i.e. the $\int v\,du$ term) using a trig identity and see what happens.)

$$\int \sec^3(x)\,dx$$

(iii) Evaluate the following indefinite integral. (Hint: Employ the strategy in the previous problem. You can use your knowledge of $\int \sec^3(x)\,dx$ to help finish solving the integral.)

$$\int \sec^5(x)\,dx$$

*(iv) [Extra Credit] Describe a process to evaluate $\int \Big[\sec(x)\Big]^{2k+1}\,dx$ where $k \geq 0$ is any integer (i.e. isolated odd powers of secant).

Wednesday: Trigonometric Substitution Act I

1)

Use the following steps to find the area enclosed by the curve $x^{2/3} + y^{2/3} = 1$ (this looks like a ninja star centered at the origin).

(a) Take advantage of the symmetry of this shape (try to determine the shape on your own instead of resorting to certain popular online computational engines) to write down a definite integral which gives the area of the ninja star.

(b) Your integral above should have an "inner" expression which is amenable to trigonometric substitution. Remember you want to end up with an expression like $1 - \cos^2(\theta)$ after substituting in order to eliminate square roots. Write the integral you get after substitution in the indefinite form (i.e. ignore bounds for now).

(c) Evaluate the indefinite integral in part (b) using the techniques from Monday/Section 7.2. Then re-substitute and evaluate at the bounds determined in part (a). This gives the area of the ninja star.

Friday: Trigonometric Substitution Act II

The following integrals can be solved using the techniques of trigonometric substitution (Wednesday/Friday lectures and Section 7.3) and/or previous techniques learned in this course. These problems are assigned for the purpose of practice as much of this week involves rote-learned pattern recognition.

P.S. The a in (iv) is to be treated as a constant.

(i)

$$\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$$

(ii)

$$\int \frac{1}{\sqrt{16 + x^2}}\, dx$$

(iii)

$$\int_{\sqrt{2}/3}^{2/3} \frac{1}{x^5\sqrt{9x^2-1}}\,dx$$

(iv)

$$\int_0^a \frac{1}{(a^2+x^2)^{3/2}}\,dx$$

(v)

$$\int \frac{1}{x\sqrt{5-x^2}}\,dx$$