H.W. 4

Due Tuesday Feb. 10th

Monday: Areas Between Curves

Find the area of the region(s) bounded by the given curves. Include a sketch with intersection points labelled.

(i)

$$x = y^3 - y$$
$$x - 3y = 0$$

(ii) (Hint: To factor, let $u = x^2$ be your new variable. Use quadratic formula then re-substitute.)

$$y = -x^4 + 5x^2 - 4$$
$$y = x^4 - 5x^2 + 4$$

Tuesday: Volumes by Cross-Section

1)

We used the idea of the infinitesimal (i.e. infinitely small but not zero) quantity dx to write down formulas for shapes which we obtained by rotating regions about an axis. Rotating gave us either disks or "washers". Taking the disk example, the typical cross-section had "infinitesimal" volume $= \pi(r(x))^2\,dx$. But, of course, not all solids have these circular cross sections.

(a) One such familiar shape is a four-sided pyramid (i.e. with a square base). Suppose we had a pyramid with the following parameters:

- 5 units in height
- sides which form an angle of $\pi/3$ with the base.

Write a formula which expresses the "infinitesimal" volume of a typical cross-section (i.e. an expression involving dx or dy), then write down a definite integral which gives the volume of the pyramid. Evaluate this integral.

(b) Now let's try a three-sided pyramid with a base in the shape of an equilateral triangle. Suppose that the top of this pyramid has been sliced off by a plane parallel to the base so that it is now $1/3$ units in height. The top is now an equilateral triangle instead of a point. Suppose the base has side length 2 and the top has side length 1.

Write a formula which expresses the "infinitesimal" volume of a typical cross-section (i.e. an expression involving dx or dy), then write down a definite integral which gives the volume of the truncated pyramid. Evaluate this integral.

Wednesday: Volumes by Cylindrical Shells and Axes of Rotation

Find the volume of the following solids using the appropriate method. Include a rough sketch.

(i) The solid formed by rotating the region in the first quadrant bounded by $y = 0$ and $y = -x^3 + x$ about the $x = 2$ axis.

(ii) The solid formed by rotating the region in the first quadrant bounded by $y = 0$ and $y = -x^3 + x$ about the $y = 2$ axis.

(iii) Let R be the region in the first quadrant bounded by $x = 0$ and $x = y\,\sin(y^3 - 1)$ that is closest to the x-axis. Form a solid by rotating R about the x-axis.