H.W. 2

Due Tuesday Jan. 20th

Monday: Fundamental Theorem of Calculus 1

Use the first part of The Fundamental Theorem of Calculus to compute the derivative of the following functions:

(i) Hint: Write $g(x) = \int_1^x ln(t)\,dt$ and $h(x) = x^4$. Then construct f from these functions.

$$f(x) = \int_1^{x^4} ln(t)\,dt$$

(ii) Hint: The problem is that the variable x appears on both sides of the limits of integration. Use a property of definite integrals to fix this.

$$f(x) = \int_{x^2}^{x^3} t^2\,dt$$

Wednesday: Fundamental Theorem of Calculus 2

1)

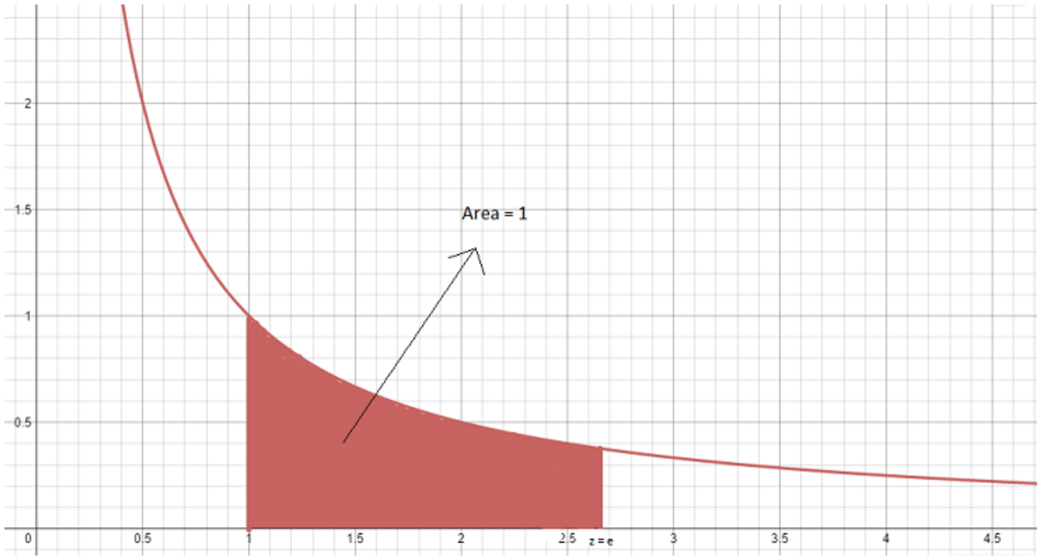
Recall that the second conclusion of the Fundamental Theorem of Calculus uses an anti-derivative to compute $\int_a^b f(x)\,dx$. By the first conclusion of the theorem, one such anti-derivative is $g(x) = \int_a^x f(t)\,dt$ (The "area from a up til x" function). If $f(x) = x^2$, then can all anti-derivatives of f be written as $F(x) = \int_a^x f(t)\,dt$ for some number a? (Hint: Recall anti-derivatives of the same function can only differ by a constant. This means you only need to know the anti-derivative at one value to determine which one it is. Keeping this in mind, try arguing that, by choosing a appropriately, you can make $F(0) = \int_a^0 t^2\,dt$ any number you want.)

2)

Can you find a familiar function $h(x)$ such that there is an anti-derivative of h that cannot be written as $H(x) = \int_a^x h(t)\,dt$ for any number a? (Hint: Find a function with an anti-derivative that is never 0.)

3)

When you're given the natural log function $ln(x)$, it is defined as the inverse of the exponential function e^x (that is $ln(e^x) = x$ and $e^{ln(x)} = x$). However, suppose we didn't know the value of e. (I mean really, how do we know the value of e anyways?) Notice that $ln(x)$ is an anti-derivative of $1/x$ (indeed, the derivative of $ln(x)$ is $1/x$). First, find an a such that $ln(x) = \int_a^x 1/t\,dt$. Then use this definition of $ln(x)$ to give a loose description of a method to approximate e. (Hint: Look at the graph of $1/x$ below. Notice that if $ln(z) = 1$, then by taking the exponential on both sides we get $z = e$. Then think about how to approximate z.)



Friday: u Substitution Trick

(i) Find an anti-derivative that could be used to compute $\int_a^b g'(x)f'(g(x))\,dx$.

(ii) Compute the following (Hint: Use the u substitution trick twice and remember to "work from the inside out".)

$$\int_0^1 (2x+1)cos(x^2+x)e^{sin(x^2+x)}\,dx$$