H.W. 1

Due Monday Jan. 12th

Monday: Math 1 review

1)

Suppose f is a differentiable function. For each situation below, say what it means about f at the point x:

(i) $f'(x) > 0$

(ii) $f'(x) < 0$

(iii) $f'(x) = 0$

2)

(i) What do we need to know about f in order to use the intermediate value theorem? Draw an example of a function on an interval for which the conclusion of the intermediate value theorem is not true.

(ii) What do we need to know about f in order to use the mean value theorem? Draw an example of a continuous function on an interval for which the conclusion of the mean value theorem is not true.

3)

(i) Suppose $f(x) = a^x$, where a is some positive number. What is $f'(x)$? (Hint: Apply the natural log function to both sides of $f(x) = a^x$ then use implicit differentiation)

(ii) Suppose $f(x) = x^x$ on the interval $(0, \infty)$. What is $f'(x)$?

Wednesday: Using rectangles to approximate areas

1)

Suppose you approximate the area under the graph of a function f on the interval $[a, b]$ using some partition of $[a, b]$ to place rectangles, as we have done in class. Geometrically speaking, why does refining the partition* give you a better approximation of the area under the graph? *(that is, using pieces of $[a, b]$ which are gotten from breaking pieces of the original partition into smaller pieces)

2)

Recall the example of velocity and position from class. As functions of time, position is the anti-derivative of velocity. We argued in class that, using a sum of rectangles and the graph of velocity, we could approximate the position function. Would using finer partitions (that is, partitions with smaller pieces) give better and better approximations of the position function? Can you write the position function in terms of the velocity function, using a definite integral?

Friday: "Eyeballing" the definite integral

1)

Recall that the definite integral $\int_a^b f(x)dx$ is the area of the region under the graph of f where f is above the x-axis minus the area above the graph of f where f is below the x-axis. Using this geometric interpretation, determine the numerical value of the following integrals:

(i) $\int_0^2 3x + 1dx$

(ii) $\int_{-2}^2 \sqrt(4 - x^2)dx$ (Hint: the graph of $\sqrt(4 - x^2)$ is what familiar geometric object?)

(iii) $\int_0^{2\pi} sin(x)dx$