Midterm 1 Review Sheet

List of Topics:

- Derivative Rules and Basic Derivatives
  - Chain Rule
  - Product Rule
  - Basic Derivatives: polynomials, trigonometric functions, $ln(x)$, e^x, $x^{n/m}$
- Rectangular Approximations (Riemann Sums)
  - Left end point (L_n)
  - Right end point (R_n)
  - Midpoint (M_n)
- Definite Integral
  - $\int_a^b f(x)\,dx$
  - Definition in term of Riemann Sums
  - Geometric Interpretation
- Fundamental Theorem of Calculus part 1
  - Statement of Theorem
  - Applications
- Fundamental Theorem of Calculus part 2
  - Statement of Theorem
  - Applications
- Indefinite Integral
  - $\int f(x)\,dx$
  - Don't forgot to add "$+C$" to your anti-derivative
- u - Substitution (reverse chain rule)
  - $du = (\frac{du}{dx})dx = u'dx$
  - Recognize when to use it
  - $u =$ the "nested" function
  - $u =$ the function whose derivative is sitting outside
  - You can use either x bounds on an anti-derivative written in terms of x, or use the u bounds on an anti-derivative written in terms of u.
- Integration by Parts (reverse product rule)
  - $\int_a^b u\,dv = uv|_a^b - \int_a^b v\,du$
  - Use it when either told to or when u - Substitution fails
  - Remember that the "parts" (u and dv) are parts of a product
  - You have a limited number of choices for what u and dv can be, sometimes recognizing a u - Substitution integral as part of the product is necessary to determine your choices for u and dv.

Representative sample of problems

Derivatives Practice:

Find the Derivative

(i)

$$f(x) = e^{x^2} sin(ln(x))$$

(ii)

$$f(x) = ln(tan(e^{(x^2+x)}))$$

(iii)

$$f(x) = \frac{arcsin(ln(x))}{e^x(x^4 - 3x^3 + x - e)}$$

(iv)

$$f(x) = ln(ln(x))e^{sin(x)} - sin(sin(sin(x)))$$

Rectangular Approximations (Riemann Sums):

Find L_n, R_n, and M_n for the given function on the given interval:

(Remember that n tells you how many equal sized pieces to break the interval into to use as the base of your rectangles)

(i) $f(x) = 2x^3 - x^2 + 1$ Interval: $[0, 3]$ $n = 3$

(ii) $f(x) = sin(x) + 1/2$ Interval: $[-2\pi, 2\pi]$ $n = 4$

(iii) $f(x) = 3x + 2$ Interval: $[-3, -2]$ $n = 5$

The definite integral:

Definition: Let f be a continuous function on the interval $[a, b]$. The definite integral of f over the interval $[a, b]$, denoted $\int_a^b f(x)\,dx$, is defined to be

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n = \lim_{n\to\infty} M_n$$

(all of the limits have the same value). Alternatively you may also, more generally speaking, break the interval into n equal sized pieces with the size denoted $\Delta x = \frac{b-a}{n}$ and choose any "sample point" from each piece of the partition, denoted x_i^* where $1 \leq i \leq n$ for each piece of the partition of the interval into n pieces, and define the definite integral as

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i^*)$$

Use the geometric interpretation of the definite integral to find the following:

(i)

$$\int_1^3 4x - 2\,dx$$

(ii)

$$\int_{-3}^{3} \sqrt{9 - t^2}\,dt$$

(iii)

$$\int_{-10}^{10} sin(\theta)\,d\theta$$

Fundamental Theorem of Calculus part 1:

Theorem: If f is a continuous function on $[a, b]$, then we can define a function g by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leq x \leq b$$

In which case g will be continuous on $[a, b]$, differentiable on (a, b) and (most importantly)

$$g'(x) = f(x).$$

Use the Fundamental Theorem of Calculus to find the derivatives of the following functions:

(i)

$$f(x) = \int_1^x ln(u)e^{u^2}\,du$$

(ii)

$$f(x) = \int_x^2 sin(ln(sec(t)))\,dt$$

(iii)

$$f(x) = \int_0^{x^5} e^{t^2}\,dt$$

(iv)

$$f(x) = \int_{x^2}^{sin(x)} e^{t^7}\,dt$$

Fundamental Theorem of Calculus part 2:

Theorem: If f is continuous on $[a, b]$ and F is any anti-derivative of f (i.e. $F' = f$) then

$$\int_a^b f(x)\,dx = F(a) - F(b)$$

Evaluate the following definite integrals:

(i)

$$\int_0^{\pi} sin(x)\,dx$$

(ii)

$$\int_0^1 e^u\,du$$

(iii)

$$\int_0^2 t^4 + 3t^2 + 5t + 2\,dt$$

(iv)

$$\int_{-\pi}^{-\pi/2} -cos(x) + \frac{1}{3x}\,dx$$

(v)

$$\int_1^3 u^{-11/8}\,du$$

(vi)

$$\int_{\pi/6}^{\pi/3} (sec(\theta))^2 - 2csc(2\theta)cot(2\theta)\,d\theta$$

Indefinite Integral:

Find expressions for the following indefinite integrals (don't forgot the "$+C$"):

(i)

$$\int \frac{2}{u}\ du$$

(ii)

$$\int \frac{7^x}{3}\ dx$$

(iii)

$$\int tan(3t+1) + t^{-10/11}\ dt$$

u - Substitution:

Evaluate the following integrals (definite and indefinite):

(i)

$$\int 7x^{-6} cos(x^{-5})\ dx$$

(ii)

$$\int \frac{1}{3} x^5 e^{2x^6}\ dx$$

(iii)

$$\int (4x^2+1)(4x^3+3x)^{2/3}\ dx$$

(iv)

$$\int_0^{\pi/4} (sin(t))^3 cos(t)\ dt$$

(v)

$$\int_{1/2}^{1} \frac{ln(2t)}{3t}\ dt$$

(vi)

$$\int_0^{\pi/4} sec^2(\theta) tan(\theta)\ dt$$

Integration by Parts:

Evaluate the following integrals (definite and indefinite):

(i)

$$\int 3^x 8^x \ dx$$

(ii)

$$\int tln(t) \ dt$$

(iii)

$$\int arcsin(x) \ dx$$

(iv)

$$\int arctan(x) \ dx$$

(v)

$$\int_0^1 t^{13} e^{4t^7} \ dt$$

(vi)

$$\int_0^{\sqrt{\pi/2}} 3t^3 sin(t^2) \ dt$$

(vii)

$$\int_{\pi/4}^{5\pi/4} e^{2x} sin(x) \ dx$$