Midterm 2 Review Sheet

List of Topics:

- Volumes/Solids of Revolution
  - Typical cross sections/Infinitesimal volume elements
  - Washer Method
  - Cylindrical Shell Method
- Trigonometry Fundamentals
  - Derivatives
  - Integrals
  - Identities:
    - $\cos^2(\theta) + \sin^2(\theta) = 1 \quad \rightarrow \quad 1 + \tan^2(\theta) = \sec^2(\theta)$
      $\cot^2\theta + 1 = \csc^2(\theta)$
    - $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \quad \rightarrow \quad \sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$
      $\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$
- Trigonometric Integrals
  - Finding $\int \cos^n(x)\sin^m(x)\,dx$
    - At least one of m or n is odd
    - Both n and m are even
  - Finding $\int \sec^n(x)\tan^m(x)\,dx$
    - n is even
    - m is odd
- Trigonometric Substitution
  - The Process:
    - Spot a square-root, but it is not a u-substitution
    - Determine the correct substitution $^{(*)(**)}$
    - Make the substitution and don't forget dx
    - Evaluate resulting trigonometric integral $^{(***)}$
    - Resubstitute to get the integral in terms of x $^{(****)}$
  - Special Considerations:
    - (*) Uneven Coefficients
    - (**) Completing the square
    - (***) Recollecting terms into a familiar form
    - (****) Triangle Trick
- Partial Fraction Decomposition
  - Three possible forms:
    - Distinct linear factors
    - Distinct irreducible quadratics
    - Repeated factors
  - Handling the irreducible quadratic term (e.g. $\int \frac{x+2}{x^2+x+1}\,dx$)
    - Split the numerator into a term amenable to u-substitution and a constant term
    - The constant term over the irreducible quadratic is handled with $\arctan(x)$

Representative sample of problems

Volumes:

1)

Find the volume of a pyramid of height 10 with a square base of side length 20 by using infinitesimal volume elements.

2)

Let R be the region bounded by the curves $y = x^2$ and $x = y^2$. Find the volume of the solid obtained by rotating R about the x-axis using first washers then cylindrical shells. Also find the volume of the solid obtained by rotating R about the y-axis using first washers then cylindrical shells.

3)

Let R be the region in the first quadrant bounded by the curves $x = 0$ and $x = \sin(\pi y)$.

(a) Find the volume of the solid obtained by rotating R about the line $y = 3$.

(b) Find the volume of the solid obtained by rotating R about the line $x = -2$.

Trigonometric Integrals:

(i)

$$\int \cos^2(3x)\sin^2(3x)\,dx$$

(ii)

$$\int \cos^7(3x)\,dx$$

(iii)

$$\int \cos^6(x)\sin^5(x)\,dx$$

(iv)

$$\int \tan^3(x)\sec^3(x)\,dx$$

(v)

$$\int \tan^4(x)\sec^8(x)\,dx$$

(vi)

$$\int \cot^5(x)\csc^3(x)\,dx$$

Trigonometric Substitution:

(i)

$$\int \frac{x^2}{(x^2+9)^{7/2}}\,dx$$

(ii)

$$\int \frac{x^3}{(4-2x^2)^{5/2}}\,dx$$

(iii)

$$\int_{2\sqrt{5}}^{2\sqrt{5/3}} \frac{x^3}{(x^2-5)^{5/2}}\,dx$$

Partial Fraction Decomposition:

1)

Write the form of the partial fraction decomposition for the following (don't bother solving for the variables A, B, C etc.)

(i)

$$\frac{x^2+2x-1}{(x-1)x^3}$$

(ii)

$$\frac{x+3}{(x^2+10)(x+1)^4}$$

(iii)

$$\frac{x^3+2}{(x^2+x+1)(x^2-4x+5)^3(x+4)}$$

2)

Evaluate the following integrals

(i)

$$\int \frac{x^2 - 2x - 2}{x(x-1)(x-3)}\, dx$$

(ii)

$$\int \frac{3x^2 + 2x + 3}{(x^2+1)(x+1)}\, dx$$

(iii)

$$\int \frac{3x^2 + 4x}{(x^2 + 2x + 2)(x+2)^2}\, dx$$