Name Solutions Date ____________

Math 2 — Quiz 3

1. Find the most general antiderivative $F(x)$ of $f(x) = x(2x^2+1)^{24}$.

$$F(x) = \int x(2x^2+1)^{24}\,dx$$

$u = 2x^2+1$

$du = 4x\,dx$

$$= \frac{1}{4}\int \underbrace{(2x^2+1)^{24}}_{u^{24}} \cdot \underbrace{4x\,dx}_{du}$$

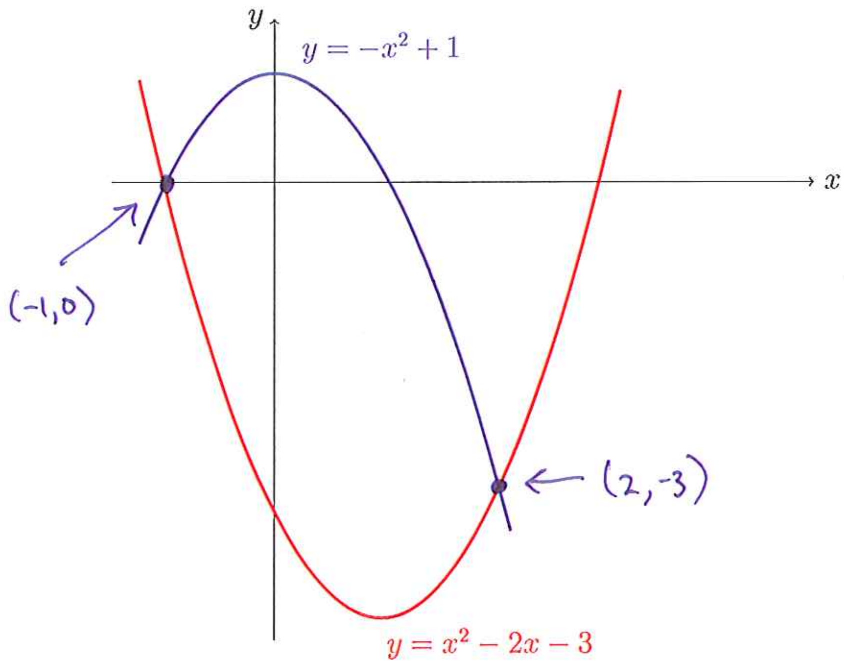
$$= \frac{1}{4}\int u^{24}\,du$$

$$= \frac{1}{4}\,\frac{u^{25}}{25} + C$$

$$= \frac{1}{100}u^{25} + C$$

$$\boxed{= \frac{1}{100}(2x^2+1)^{25} + C}$$

2. Compute the area between (or bounded by) the two curves.



First, we find the two points of intersection: Solve $-x^2+1 = x^2-2x-3$ for x.

$$\Rightarrow 2x^2-2x-4=0 \Rightarrow 2(x^2-x-2)=0$$

$$\Rightarrow 2(x-2)(x+1)=0 \Rightarrow x=2,-1.$$

Then, setup the integral & solve.

$$\int_{-1}^{2} (-x^2+1)-(x^2-2x-3)\,dx = \int_{-1}^{2} -2x^2+2x+4\,dx$$

$$= \frac{-2}{3}x^3+x^2+4x \Big|_{-1}^{2}$$

$$= \frac{-2}{3}(2)^3+(2)^2+4(2)-\left(\frac{-2}{3}(-1)^3+(-1)^2+4(-1)\right)$$

$$= \frac{-16}{3}+4+8-\frac{2}{3}-1+4$$

$$= \frac{-18}{3}+12+3 = -6+15 = \boxed{9}$$