Math 2 — Practice "Exam" 3

This is a collection of problems not covered by the first two practice exams. In other words, these problems cover Sections 7.4 (Integration by Partial Fractions), 7.8 (Improper Integrals), 8.1 (Arc Length), and 8.5 (Probability).

IMPORTANT: *The final exam is cumulative!* It is highly recommended that you review the first two practice exams in addition to these problems.

1. Use partial fractions to compute the integral $\displaystyle\int \frac{1}{x^2-1}\,\mathrm{d}x$.

2. Compute the arc length of the function $y = 8x + 1$ from $x = 0$ to $x = 1$. Is there an easier (read: geometric) way to solve this problem?

3. Why does $\displaystyle\int_{-\infty}^{\infty} \cos(\pi x)\,\mathrm{d}x$ diverge despite the fact that it doesn't tend to infinity?

4. Set up the integral to compute the arc length of $y = \ln\left|1 - x^2\right|$ from $x = 0$ to $x = 1/2$. Simplify the integral as much as you can, but **do not solve**.

5. Compute the expected value of the probability distribution function

$$p(x) = \begin{cases} 1/5, & \text{if } 1 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

6. Given the probability distribution function below, what is the probability that our value is measured in the interval $5/2 \le x \le 5$?

$$p(x) = \begin{cases} \dfrac{\pi}{20}\sin\left(\dfrac{\pi x}{10}\right), & \text{if } 0 \le x \le 10 \\ 0, & \text{otherwise.} \end{cases}$$

7. Determine whether the improper integral converges or not. If convergent, determine its value.

$$\int_{-\infty}^{\infty} x e^{-x^2}\,\mathrm{d}x.$$

8. Find the value for the constant c such that the following function is a probability distribution function

$$p(x) = \begin{cases} cx^2, & \text{if } 0 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

9. Compute the expected value of the probability distribution function

$$p(x) = \begin{cases} 4e^{-4x}, & \text{if } x \ge 0 \\ 0, & \text{otherwise.} \end{cases}$$

10. Does the following integral converge or diverge? If convergent, determine its value.

$$\int_1^\infty \frac{1}{x^2+x}\,\mathrm{d}x.$$

11. Compute the following integral $\displaystyle\int \frac{x^2+x-1}{x^3-x}\,\mathrm{d}x$.

12. Does the following integral converge or diverge? If convergent, determine its value.

$$\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x.$$

13. Which of the following integrals represents the arc length of the curve $y=\cos x$ from $x=0$ to $x=2\pi$?

(a) $\displaystyle\int_0^{2\pi} \sqrt{1+\sin x}\,\mathrm{d}x$ (b) $\displaystyle\int_0^{2\pi} \sqrt{1-\cos^2 x}\,\mathrm{d}x$. (c) $\displaystyle\int_0^{2\pi} \sqrt{2-\cos^2 x}\,\mathrm{d}x$.

14. Compute the arc length of the curve from $x=1$ to $x=4$ given by $f(x)=\displaystyle\int_1^x \sqrt{t^3-1}\,\mathrm{d}t$.

15. Set up the integral to compute the arc length of $y=\ln\big|\cos x\big|$ from $x=0$ to $x=\pi/3$. Simplify the integral as much as you can, but **do not solve**.

16. Use partial fractions to compute the integral $\displaystyle\int \frac{5x+1}{2x^2-x-1}\,\mathrm{d}x$.

17. Verify that $p(x)=\dfrac{1}{\pi(1+x^2)}$ is a probability distribution function and compute

$$\int_{-1}^1 \frac{1}{\pi(1+x^2)}\,\mathrm{d}x.$$

18. Decompose the following rational function

$$\frac{-x+5}{(x-2)^2}.$$

19. Decompose the following rational function

$$\frac{-3}{x^3-x^2+2x-2}.$$

20. Compute the value of the convergent improper integral $\displaystyle\int_0^\infty \frac{1}{(x+1)^2}\,\mathrm{d}x$.