Math 2 — Practice Exam 2

This is a practice “exam” in the sense that this is not intended to be completed in two hours. This is merely a collection of problems that I’ve cobbled together to give you a sense of what you could realistically expect to show up on the exam.

These problems range over Sections 6.3 and 6.5, as well as Sections 7.1 through 7.3, involving computing volumes with cylindrical shells (6.3), computing averages of functions (6.5), integration by parts (7.1), trigonometric integration (7.2), and trigonometric substitution (7.3). Much like on the exam, there is no indication which technique of integration you should use to solve these problems. It will be up to you to choose the correct technique and apply it correctly.

1. Compute $\displaystyle\int_0^{\pi/4} \cos^4(x)\,\mathrm{d}x$.
   $\big($Hint: remember that $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$, or use the reduction formula on pg. 469, Problem 48 in your textbook$\big)$.
2. Compute the volume of the solid generated by rotating the region bounded by $y=\sin(x)$, $y=0$, and $x=\pi/2$ about the y-axis using cylindrical shells.
3. Compute $\displaystyle\int \sin(3x)\sin(6x)\,\mathrm{d}x$. (Hint: remember that $\sin(2x) = 2\sin(x)\cos(x)$).
4. Compute $\displaystyle\int x\cos(\pi x)\,\mathrm{d}x$.
5. (a) Compute $\displaystyle\int_0^2 \sqrt{4-x^2}\,\mathrm{d}x$. (Hint: remember the hint from Problem 1).
   (b) Is there a simpler way of solving Part (a)?
   (Hint: draw the graph and interpret the integral).
6. Compute $\displaystyle\int \tan^3(x)\sec^3(x)\,\mathrm{d}x$.
7. Compute $\displaystyle\int \arcsin(x)\,\mathrm{d}x$.
   $\big($Hint: use integration by parts and remember that $\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}\big)$.
8. Compute $\displaystyle\int_0^{\pi} e^{\cos(2t)}\sin(t)\cos(t)\,\mathrm{d}t$. (Hint: remember the hint from Problem 3).
9. Compute $\displaystyle\int_0^6 \frac{\arcsin(\frac{x}{6})}{\sqrt{36-x^2}}\,\mathrm{d}x$.
10. What substitution would you make for the following integrals? Perform the substitution and simplify as much as you can, but **do not solve** (unless you want to).

    (a) $\displaystyle\int \frac{t^5}{\sqrt{t^2+5}}\,\mathrm{d}t$, (b) $\displaystyle\int \frac{1}{x^5\sqrt{9x^2-1}}\,\mathrm{d}x$, (c) $\displaystyle\int \frac{1}{x\sqrt{5-x^2}}\,\mathrm{d}x$.

11. Compute $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,\mathrm{d}x$.

12. Compute the average of $f(x) = x\sin(x^2)$ on the interval $[\sqrt{\pi}, 2\sqrt{\pi}]$.

13. Compute the volume of the solid generated by rotating the region bounded by $y = 1/x^2$, $y = 0$, $x = 1$, and $x = e^2$ about the y-axis using cylindrical shells.

14. Compute $\displaystyle\int \frac{\sqrt{\arctan(x)}}{1+x^2}\,\mathrm{d}x$.

15. Compute the average of $f(x) = 1/x$ on the interval $[1, 3]$.

16. Compute $\displaystyle\int \cos^2(x)\sin(2x)\,\mathrm{d}x$. (Hint: remember the hint from Problem 3).

17. Compute $\displaystyle\int_0^2 x^2\sqrt{4-x^2}\,\mathrm{d}x$.

18. Compute $\displaystyle\int_0^1 (x^2+1)e^{-x}\,\mathrm{d}x$.

19. Compute $\displaystyle\int_0^{\pi/4} \tan^4(x)\,\mathrm{d}x$.

20. Compute $\displaystyle\int_1^4 \frac{\ln|x|}{\sqrt{x}}\,\mathrm{d}x$.

21. (a) Show that $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\,\mathrm{d}x = \tan(\arcsin(\tfrac{x}{2})) - \arcsin(\tfrac{x}{2}) + C$.

    (b) Simplify $\tan(\arcsin(\frac{x}{2}))$.

    (c) Show that $\displaystyle\int_0^1 \frac{x^2}{(4-x^2)^{3/2}}\,\mathrm{d}x = \frac{2\sqrt{3}-\pi}{6}$.

22. Compute $\displaystyle\int \sec^6(x)\,\mathrm{d}x$.