Math 2 — Practice "Exam" 1

This is a practice "exam" in the sense that this is not intended to be completed in two hours. This is merely a collection of problems that I've cobbled together to give you a sense of what you could realistically expect to show up on the exam.

1 Section 5.1 — Areas and Distances

1. (a) Estimate the area under the graph of $f(x) = \sqrt{x}$ from $x = 0$ to $x = 4$ using four rectangles with the left endpoint method.
(b) Is this an overestimate or underestimate?
(c) What property of the function $f(x)$ allows us to make the claim from part (b)?

2 Section 5.2 — The Definite Integral

2. Given that $\displaystyle\int_0^{\pi} \sin^2 x \, \mathrm{d}x = \frac{\pi}{2}$, compute $\displaystyle\int_0^{\pi} (4\sin^2 t + 7) \, \mathrm{d}t$.

3. Use **geometry** to evaluate the definite integral $\displaystyle\int_2^5 (4 - 2x) \, \mathrm{d}x$.

4. Use **geometry** to evaluate the definite integral $\displaystyle\int_2^{10} \left|x - 5\right| \, \mathrm{d}x$.

5. If $\displaystyle\int_0^9 f(x) \, \mathrm{d}x = 37$ and $\displaystyle\int_0^9 g(x) \, \mathrm{d}x = 16$, find $\displaystyle\int_0^9 (2f(x) + 3g(x)) \, \mathrm{d}x$.

3 Section 5.3 — Fundamental Theorem of Calculus

6. (a) State the First Fundamental Theorem of Calculus.
(b) Interpret the First Fundamental Theorem of Calculus in terms of antiderivatives and derivatives of a function.
(c) State the Second Fundamental Theorem of Calculus.
(d) Interpret the Second Fundamental Theorem of Calculus in terms of geometry.

7. Use the Chain Rule to evaluate $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x} \int_0^{x^2} e^t \, \mathrm{d}t$.

8. Evaluate the following integrals:

(a) $\displaystyle\int_1^8 x^{-2/3} \, \mathrm{d}x$ (b) $\displaystyle\int_0^4 (4 - t)\sqrt{t} \, \mathrm{d}t$ (c) $\displaystyle\int_1^9 \sqrt{\frac{3}{z}} \, \mathrm{d}z$

9. Given $F(x) = \int_x^0 \frac{e^t}{t}\,\mathrm{d}t$, find $F'(x)$.

4 Section 5.4 — The Indefinite Integral

10. Find the most general antiderivative $F(x)$ of $f(x) = x^2 - x^{-2}$.

11. Verify with derivatives that $\int \cos^3 x\,\mathrm{d}x = \sin x - \frac{1}{3}\sin^3 x + C$.

12. Evaluate the following indefinite integrals:

(a) $\int (1 + \tan^2 u)\,\mathrm{d}u$ (b) $\int \frac{x^3 - 2\sqrt{x}}{x}\,\mathrm{d}x$ (c) $\int \frac{t^4 - 1}{t^2 - 1}\,\mathrm{d}t$.

5 Section 5.5 — The Substitution Rule

13. Evaluate the following indefinite integrals:

(a) $\int \frac{e^u}{(1 - e^u)^2}\,\mathrm{d}u$ (b) $\int \sqrt{x}\sin(1 + x^{3/2})\,\mathrm{d}x$ (c) $\int x\cos(x^2)\sin(x^2)\,\mathrm{d}x$.

14. Evaluate the following definite integrals:

(a) $\int_0^1 \cos\left(\frac{\pi t}{2}\right)\,\mathrm{d}t$ (b) $\int_0^1 (3t - 1)^{50}\,\mathrm{d}t$ (c) $\int_0^3 \frac{1}{5x + 1}\,\mathrm{d}x$.

6 Section 6.1 — Areas Between Curves

15. Compute the area bounded by $y = |x|$ and $y = x^2 - 2$.

16. Compute the area bounded by $y = x^2$ and $y = 4x - x^2$.

17. Compute the area bounded by $y = \sqrt{x + 2}$ and $y = \frac{1}{x+1}$ between $x = 0$ and $x = 2$.

7 Section 6.2 — Volumes

18. Set up the integral—but **do not solve**—to find the volume of the solid obtained by rotating the region bounded by $y = e^{-x^2}$, $y = 0$, $x = -1$, and $x = 1$ about the x-axis.

19. Compute the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the line $y = 1$.

20. Compute the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, $x = 1$ about the line $x = 2$.