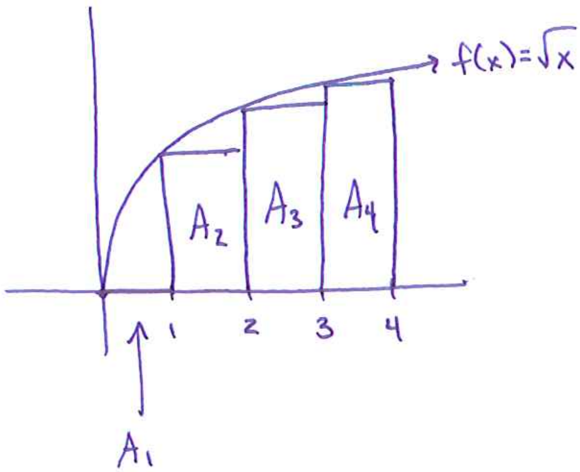
1. (a)



Using left endpoints, we have the following four rectangles.

$A_1 = 0$

$A_2 = 1 \cdot f(1) = \sqrt{1} = 1$

$A_3 = 1 \cdot f(2) = \sqrt{2} = \sqrt{2}$

$A_4 = 1 \cdot f(3) = \sqrt{3}$

Then, an approximate area is $0 + 1 + \sqrt{2} + \sqrt{3} = \boxed{1+\sqrt{2}+\sqrt{3}} \approx 4.146$

The true area under the curve is $\int_0^4 \sqrt{x}\,dx = \frac{16}{3} \approx 5.33$

(b) This is an $\boxed{\text{underestimate}}$ by looking at the graph from part (a).

(c) The function $\sqrt{x}$ is $\boxed{\text{increasing}}$ and so we get an underestimate.

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2. $\int_0^{\pi} \sin^2 x\,dx = \frac{\pi}{2}$, so $\int_0^{\pi} 4\sin^2 t + 7\,dt = 4\int_0^{\pi} \sin^2 t\,dt + 7\int_0^{\pi} dt$

$$= 4\left(\frac{\pi}{2}\right) + 7\pi = 2\pi + 7\pi = \boxed{9\pi}$$

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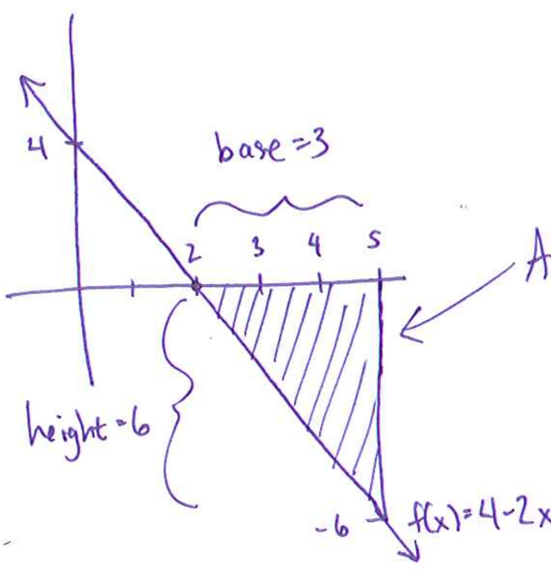
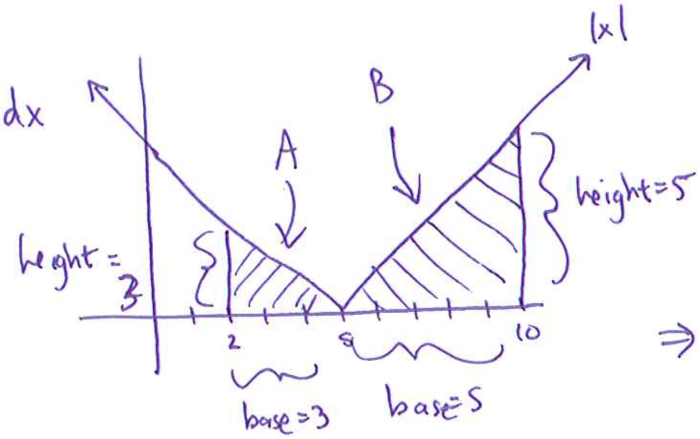
3. $\int_2^5 4-2x\,dx$



Area $= \frac{1}{2}(3)(6) = 9$, but this is below the x-axis, so

$$\int_2^5 4-2x\,dx = \boxed{-9}$$

4. $\int_2^{10} |x-5|\,dx$



$A = \frac{1}{2}(3)(3) = \frac{9}{2}$

$B = \frac{1}{2}(5)(5) = \frac{25}{2}$

$\Rightarrow \int_2^{10} |x-5|\,dx = \frac{9}{2} + \frac{25}{2} = \frac{34}{2} = \boxed{17}$

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5. If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)\,dx = 16$, then

$$\int_0^9 2f(x) + 3g(x)\,dx = 2\int_0^9 f(x)dx + 3\int_0^9 g(x)\,dx = 2(37) + \overset{3}{\cancel{2}}(16)$$

$$= 74 + 48 = \boxed{122}$$

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6. FTC 1 $\rightarrow$ For a continuous function $f(x)$, define $g(x) = \int_a^x f(t)dt$,

(a) then $\frac{d}{dx}(g(x)) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

(b). This tells us that integration (or antidifferentiation) and differentiation are inverse operations.

(c) FTC2 $\rightarrow$ For a continuous function $f(x)$ on an interval $[a,b]$, we have that $\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$.

(d) This allows us to compute the "area under the function" $f(x)$ where $f(x) \geq 0$. Otherwise, this gives us the "net area" under the curve $f(x)$ when it is not necessarily positive (i.e. $f(x)$ can have negative values.).

7. $\frac{d}{dx}\int_0^{x^2} e^t\,dt$. Define $f(x)=\int_0^x e^t\,dt$, so $\underline{f'(x)=e^x}$.

Then, $\int_0^{x^2} e^t\,dt = f(x^2)$

So, $\frac{d}{dx}\int_0^{x^2} e^t\,dt = \frac{d}{dx}\left(f(x^2)\right) = f'(x^2)\cdot 2x = e^{x^2}\cdot 2x = \boxed{2xe^{x^2}}$

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8. (a) $\int_1^8 x^{-2/3}\,dx = \frac{x^{1/3}}{1/3}\Big|_1^8 = 3\left(8^{1/3}-1^{1/3}\right) = 3(2-1) = \boxed{3}$

(b) $\int_0^4 (4-t)\sqrt{t}\,dt = \int_0^4 4t^{1/2}-t^{3/2}\,dt = 4\cdot\frac{t^{3/2}}{3/2} - \frac{t^{5/2}}{5/2}\Big|_0^4$

why?

$= 4\left(\frac{2}{3}\right)\cdot 4^{3/2} - \frac{2}{5}\cdot 4^{5/2} - (0)$

$= \frac{8}{3}\cdot 8 - \frac{2}{5}\cdot 32 = \boxed{\frac{64}{3} - \frac{64}{5}}$ ~~$= \frac{5\cdot 64 - 3\cdot 64}{15} = \ldots 48$~~

(c) $\int_1^9 \sqrt{\frac{3}{z}}\,dz = \sqrt{3}\int_1^9 z^{-1/2}\,dz = \sqrt{3}\left(\frac{z^{1/2}}{1/2}\Big|_1^9\right) = \sqrt{3}\,(2)\left(9^{1/2}-1^{1/2}\right) = 2\sqrt{3}(3-1)$

$= \boxed{4\sqrt{3}}$

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9. $F(x)=\int_x^0 \frac{e^t}{t}\,dt$, then $F'(x) = \frac{d}{dx}\int_x^0 \frac{e^t}{t}\,dt = -\frac{d}{dx}\int_0^x \frac{e^t}{t}\,dt = \boxed{-\frac{e^x}{x}}$

---

10. antiderivative of x^2-x^{-2} is $\int x^2 - x^{-2}\,dx = \frac{1}{3}x^3 - \frac{x^{-1}}{-1} + C$

$= \boxed{\frac{1}{3}x^3 + \frac{1}{x} + C}$

11. Verify $\int \cos^3 x \, dx = \sin x - \frac{1}{3}\sin^3 x + C$

Compute $\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x + C\right) = \cos x - \frac{1}{3}\cdot 3 \cdot \sin^2 x \cdot \cos x + 0$

$= \cos x - \cos x \cdot \sin^2 x$

$= \cos x \left(1 - \sin^2 x\right) = \cos x \cdot \cos^2 x = \boxed{\cos^3 x}$ ✓

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12. (a) $\int 1 + \tan^2 u \, du$

Remember that $\sin^2 u + \cos^2 u = 1$, so

$\underline{\tan^2 u + 1 = \sec^2 u}$

$= \int \sec^2 u \, du = \boxed{\tan u + C}$

~~(b) $\int \frac{x^3 - 2\sqrt{x}}{x} dx = \int x^2 - 2x^{1/2-1} dx = \int x^2 - 2x^{-1/2}$~~

(b) $\int \frac{x^3 - 2\sqrt{x}}{x} dx = \int x^2 - 2x^{-1/2} dx = \frac{1}{3}x^3 - 2 \cdot \frac{x^{1/2}}{1/2} + C = \boxed{\frac{1}{3}x^3 - 4x^{1/2} + C}$

(c) $\int \frac{t^4 - 1}{t^2 - 1} dt = \int \frac{(t^2-1)(t^2+1)}{t^2-1} dt = \int t^2 + 1 \, dt = \boxed{\frac{1}{3}t^3 + t + C}$

---

13. (a) $\int \frac{e^u}{(1-e^u)^2} du$

$t = 1 - e^u$
$dt = -e^u du$

$\Rightarrow -\int \frac{1}{t^2} dt = -\int t^{-2} dt = -\left(\frac{t^{-1}}{-1}\right) + C = \boxed{\frac{1}{t} + C}$

$\boxed{\frac{1}{1-e^u} + C}$

(b) $\int \sqrt{x} \cdot \sin\left(1 + x^{3/2}\right) dx$

$u = 1 + x^{3/2}$
$du = \frac{3}{2}x^{1/2} dx$

$\frac{2}{3}\int \sin u \, du = \frac{2}{3}(-\cos u) + C = \boxed{\frac{-2}{3}\cos\left(1 + x^{3/2}\right) + C}$

13.(c) $\int x\cos(x^2)\sin(x^2)\,dx$ $\qquad u=x^2$, $du=2x\,dx$

$\frac{1}{2}\int \cos u\cdot \sin u\,du$ $\qquad t=\sin u$, $dt=\cos u\,du$

$\frac{1}{2}\int t\,dt = \frac{1}{2}\left(\frac{1}{2}t^2\right)+C = \frac{1}{4}\sin^2 u + C = \boxed{\frac{1}{4}\sin^2(x^2)+C}$

14.(a) $\int_0^1 \cos\left(\frac{\pi t}{2}\right)dt$ $\qquad u=\frac{\pi t}{2}$ $\quad du=\frac{\pi}{2}\,dt$

$\frac{2}{\pi}\int_0^{\pi/2}\cos u\,du = \frac{2}{\pi}\left(\sin u\Big|_0^{\pi/2}\right) = \frac{2}{\pi}\left(\sin\left(\frac{\pi}{2}\right)-\sin(0)\right) = \frac{2}{\pi}(1-0) = \boxed{\frac{2}{\pi}}$

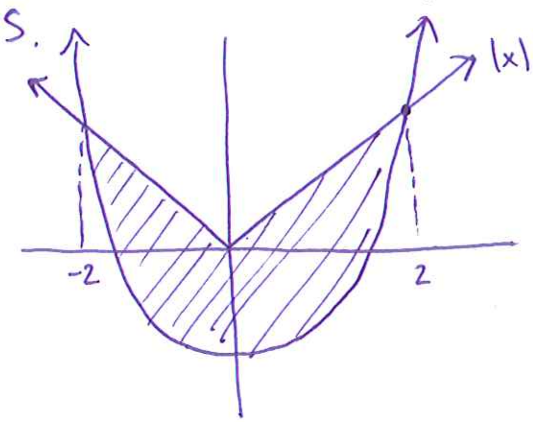
(b) $\int_0^1 (3t-1)^{50}\,dt$ $\qquad u=3t-1$, $du=3\,dt$

$\frac{1}{3}\int_{-1}^{2} u^{50}\,du = \frac{1}{3}\left(\frac{1}{51}\cdot u^{51}\Big|_{-1}^{2}\right) = \frac{1}{153}\left(2^{51}-(-1)^{51}\right) = \boxed{\frac{1}{153}\left(2^{51}+1\right)}$

(c) $\int_0^3 \frac{1}{5x+1}\,dx$ $\qquad u=5x+1$, $du=5\,dx$

$\frac{1}{5}\int_1^{16}\frac{1}{u}\,du = \frac{1}{5}\left(\ln|u|\Big|_1^{16}\right) = \frac{1}{5}\left(\ln(16)-\ln(1)\right) = \boxed{\frac{1}{5}\ln(16)}$

15.



$x^2-2=x \Rightarrow x^2-x-2=0$

$\Rightarrow (x-2)(x+1)=0$

$\Rightarrow \boxed{x=2}$, $\cancel{x=-1}$

why?

$\text{Area} = \int_{-2}^{2} |x|-(x^2-2)\,dx$

$= \int_{-2}^{2}|x|\,dx - \int_{-2}^{2}x^2-2\,dx$

why? $= 2\int_0^2 x\,dx - \left(\frac{1}{3}x^3-2x\Big|_{-2}^{2}\right)$

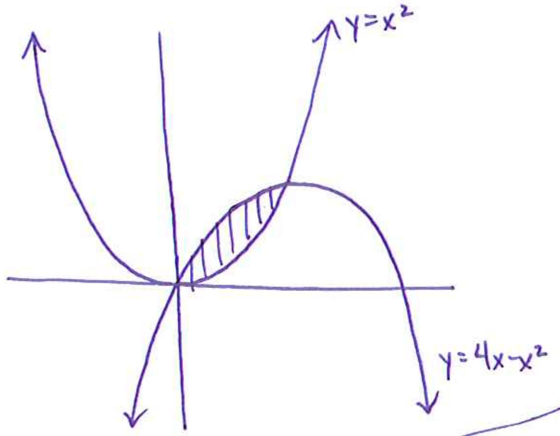
$= 2\left(\frac{1}{2}x^2\Big|_0^2\right) - \left(\frac{1}{3}(2)^3-2(2)-\left(\frac{1}{3}(-2)^3-2(-2)\right)\right)$

$= 2\left(\frac{1}{2}(2)^2-\frac{1}{2}(0)^2\right) - \left(\frac{8}{3}-4-\left(-\frac{8}{3}+4\right)\right)$

$= 2(2)-\left(\frac{16}{3}-8\right) = 4+8-\frac{16}{3} = 12-\frac{16}{3}$

$= \frac{36-16}{3} = \boxed{\frac{20}{3}}$

16. Compute area bounded by $y=x^2$ and $y=4x-x^2$.



$$\text{Area} = \int_0^2 (4x-x^2)-x^2\,dx$$

$$= \int_0^2 -2x^2+4x\,dx$$

why?

$$= -\frac{2}{3}x^3+2x^2\Big|_0^2 = -\frac{2}{3}(2)^3+2(2)^2-0$$

$$= \frac{-16}{3}+8 = \frac{-16}{3}+\frac{24}{3} = \boxed{\frac{8}{3}}$$

$$x^2=4x-x^2 \Rightarrow 2x^2-4x=0$$

$$\Rightarrow 2x(x-2)=0$$

$$\Rightarrow x=0,\ x=2$$

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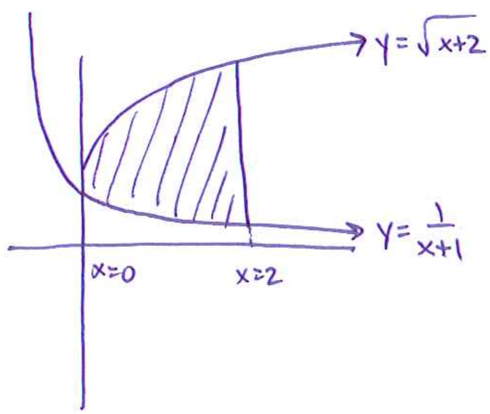
17. Compute area bounded by $y=\sqrt{x+2}$ and $y=\frac{1}{x+1}$ ~~from~~ between $x=0$ & $x=4$



$$\text{Area} = \int_0^2 \sqrt{x+2} - \frac{1}{x+1}\,dx$$

$$= \int_0^2 \sqrt{x+2}\,dx - \int_0^2 \frac{1}{x+1}\,dx$$

$$= \frac{16}{3}-\frac{4}{3}\sqrt{2} - \left(\ln|3|\right)$$

$$= \boxed{\frac{16}{3}-\frac{4}{3}\sqrt{2}-\ln(3)} \leftarrow \text{Answer}$$

Substitution Rule

$$\int_0^2 \sqrt{x+2}\,dx \qquad u=x+2,\quad du=dx$$

$$\int_2^4 \sqrt{u}\,du = \frac{u^{3/2}}{3/2}\Big|_2^4 = \frac{2}{3}\left(8-2\sqrt{2}\right)$$

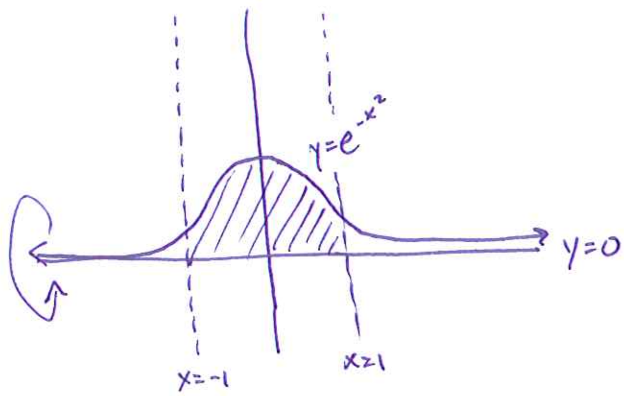
$$= \boxed{\frac{16}{3}-\frac{4}{3}\sqrt{2}}$$

$$\int_0^2 \frac{1}{x+1}\,dx \qquad u=x+1,\quad du=dx$$

$$\int_1^3 \frac{1}{u}\,du = \ln|u|\Big|_1^3 = \ln(3)-\ln(1) = \boxed{\ln(3)}$$

18. Setup the integral to rotate region bounded by

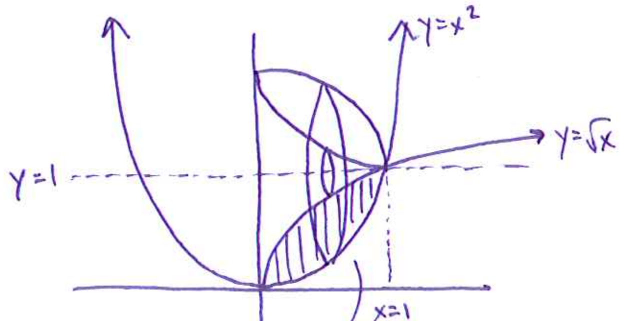
$y = e^{-x^2}$, $y = 0$, $x = -1$, and $x = 1$ about x-axis.



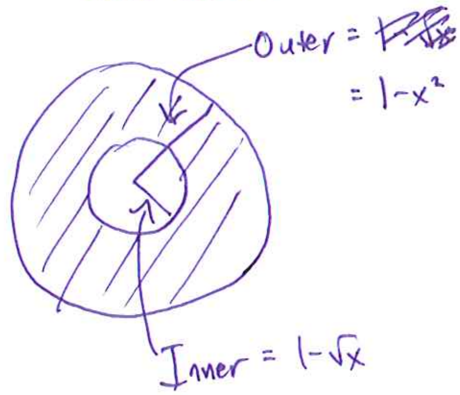
$$\text{Volume} = \int_{-1}^{1} \pi \left(e^{-x^2}\right)^2 dx$$

$$= \int_{-1}^{1} \pi e^{-2x^2} dx$$

19. Compute volume of solid obtained by rotating $y = x^2$ and $y = \sqrt{x}$ about $y = 1$.



Cross-Section



$$\text{Volume} = \int_0^1 \pi\,(\text{Outer})^2 - \pi\,(\text{Inner})^2\, dx$$

$$= \pi \int_0^1 (1-x^2)^2 - (1-\sqrt{x})^2\, dx$$

$$= \pi \int_0^1 1 - 2x^2 + x^4 - (1 - 2\sqrt{x} + x)\, dx$$

$$= \pi \int_0^1 -2x^2 + x^4 + 2x^{1/2} - x\, dx$$

$$= \pi \left(\frac{-2}{3}x^3 + \frac{1}{5}x^5 + 2 \cdot \frac{x^{3/2}}{3/2} - \frac{1}{2}x^2 \Big|_0^1 \right)$$

$$= \pi \left(\frac{-2}{3} + \frac{1}{5} + \frac{4}{3} - \frac{1}{2} \right)$$

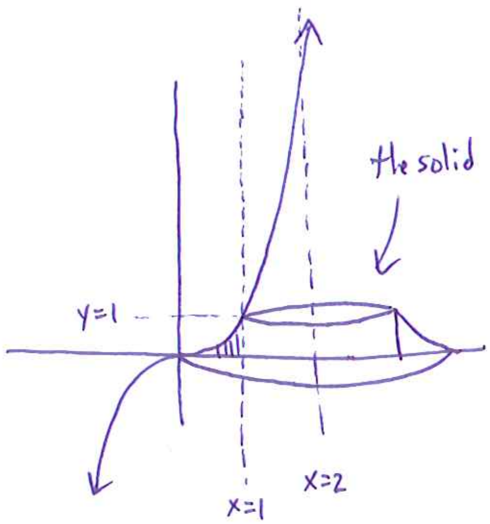
$$= \pi \left(\frac{2}{3} + \frac{1}{5} - \frac{1}{2} \right)$$

$$= \pi \left(\frac{20}{30} + \frac{6}{30} - \frac{15}{30} \right)$$

$$= \boxed{\frac{11\pi}{30}}$$

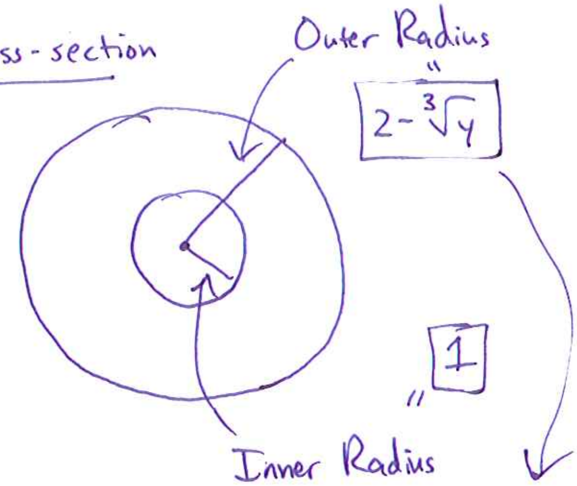
20. Compute volume of solid generated by rotating $y=x^3$, $y=0$, $x=1$ about line $x=2$.

~~Solve $x^3=0 \Rightarrow x=0$~~



Endpoints come from $y=0$ to $y=1$

Cross-section



The center of the cross-section occurs where $x=2$

The outer part occurs where $x=\sqrt[3]{y} \Rightarrow \boxed{\text{Outer} = 2-\sqrt[3]{y}}$

$$\text{Volume} = \int_0^1 \pi(\text{Outer})^2 - \pi(\text{Inner})^2 \, dy$$

$$= \int_0^1 \pi\left(2-\sqrt[3]{y}\right)^2 - \pi(1)^2 \, dy$$

$$= \pi \int_0^1 4 - 4y^{1/3} + y^{2/3} - 1 \, dy$$

$$= \pi \int_0^1 3 - 4y^{1/3} + y^{2/3} \, dy$$

$$= \pi \left(3y - 4 \cdot \frac{y^{4/3}}{4/3} + \frac{y^{5/3}}{5/3} \Big|_0^1 \right)$$

$$= \pi\left(3 - 4\cdot\frac{3}{4} + \frac{3}{5}\right) = \pi\left(3 - 3 + \frac{3}{5}\right) = \boxed{\frac{3\pi}{5}}$$