Quiz 6: Improper Integrals and Arc Length

March 6, 2013

Name: Key Section:

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Determine if the integral $\int_0^\infty \frac{1}{\sqrt{1+x}}\,dx$ is convergent or divergent. If convergent evaluate the integral.

$$\int_0^\infty \frac{1}{\sqrt{1+x}}\,dx = \lim_{t\to\infty} \int_0^t (1+x)^{-1/2}\,dx$$

$$= \lim_{t\to\infty} 2(1+x)^{1/2}\Big|_0^t$$

$$= \lim_{t\to\infty} \underbrace{2\sqrt{1+t}}_{\to\infty} - 2$$

DIVERGES

2. Find the arc length of the curve $y = 1 + 2x^{3/2}$, $0 \le x \le 1$

Arc Length $= \int_a^b \sqrt{1+(f'(x))^2}\, dx$

$\frac{dy}{dx} = 2\cdot\left(\frac{3}{2}\right)x^{1/2} = 3\sqrt{x}$

Arc Length $= \int_0^1 \sqrt{1+(3\sqrt{x})^2}\, dx$

$= \int_0^1 \sqrt{1+9x}\, dx$ $\quad u = 1+9x,\ du = 9dx$

$= \int_0^1 \frac{1}{9}\sqrt{u}\, du$

$= \frac{1}{9}\left(\frac{2}{3}\right)u^{3/2}\Big|_0^1$

$= \frac{2}{27}(1+9x)^{3/2}\Big|_0^1$

$= \frac{2}{27}(10^{3/2}-1)$

$= \boxed{\frac{2}{27}(10\sqrt{10}-1)}$