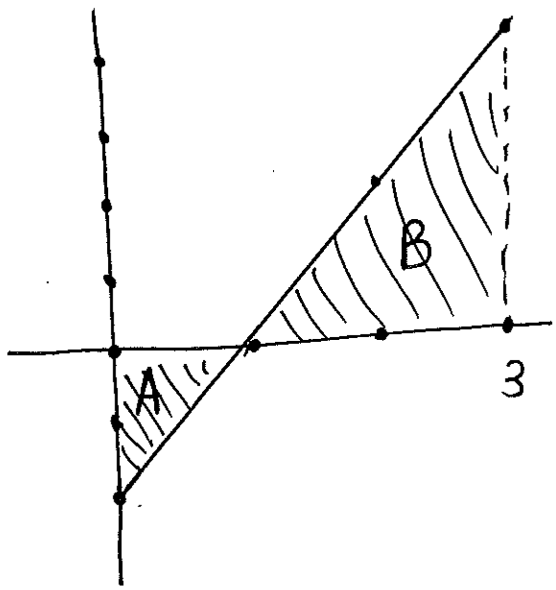
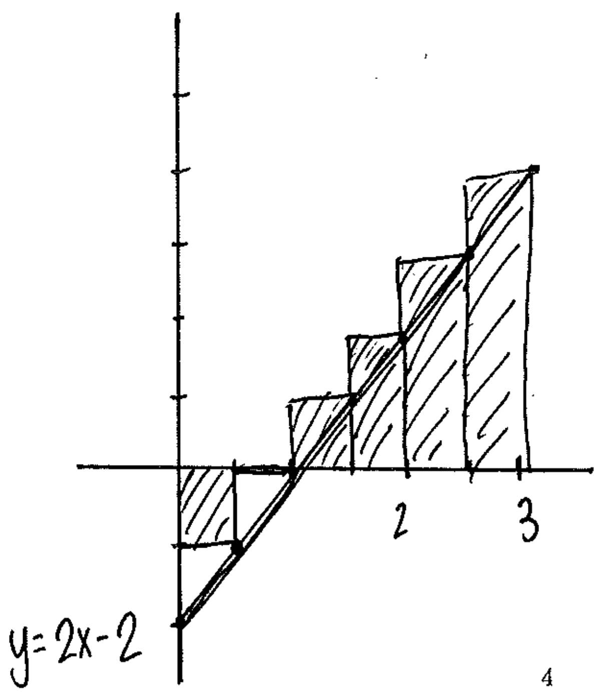
1. In this problem you will compute the integral $\int_0^3 2x - 2\,dx$ in three different ways.

   (a) (6 pts) Evaluate the integral by interpreting it in terms of area. Start by drawing the graph of the function $f(x) = 2x - 2$ on the interval $[0, 3]$.



$B - A = \frac{1}{2}\cdot 2\cdot 4 - \frac{1}{2}\cdot 2\cdot 1$
$= 4 - 1 = \boxed{3}$

   (b) (6 pts) Approximate the integral using a Riemann sum with six rectangles (n=6) and right endpoints. Is this an under-approximation or an over-approximation?



$R_6 = (2\cdot\frac{1}{2}-2)\frac{1}{2} + (2\cdot 1-2)\frac{1}{2} + (2\cdot\frac{3}{2}-2)\frac{1}{2}$
$+ (2\cdot 2-2)\frac{1}{2} + (2\cdot\frac{5}{2}-2)\frac{1}{2} + (2\cdot 3-2)\frac{1}{2}$
$= \frac{1}{2}(-1+0+1+2+3+4)$
$= \boxed{9/2}$

Over-approximation

(c) (5 pts) Evaluate the integral using Part 2 of the Fundamental Theorem of Calculus.

$$\int_0^3 2x - 2 \, dx = x^2 - 2x \Big|_0^3$$

$$= 9 - 6 - (0 - 0) = 3$$

2. (5 pts) Evaluate the integral $\int \frac{1}{x \cdot (\ln x)^3} \, dx$ using the substitution $u = \ln x$.

$u = \ln x$

$du = \frac{1}{x} \, dx$

$$\int \frac{1}{x (\ln x)^3} \, dx = \int \frac{1}{u^3} \, du$$

$$= -\frac{1}{2} u^{-2} + C$$

$$= \frac{-1}{2 (\ln x)^2} + C$$

3. In this question you will state both parts of the Fundamental Theorem of Calculus:

Suppose $f(x)$ is continuous on $[a, b]$.

(4 pts) Part 1:

If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$

(4 pts) Part 2:

$\int_a^b f(x)\,dx = F(x)\Big|_a^b$ where $F'(x) = f(x)$

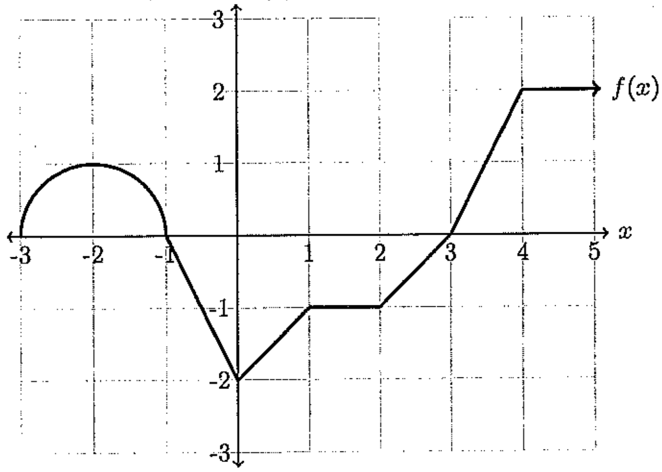
i.e., F is any antiderivative of f.

4. (5 pts) If $g(x) = \int_x^2 \frac{\ln t}{t}\,dt$ find $g'(x)$.

$g(x) = -\int_2^x \frac{\ln t}{t}\,dt$

$g'(x) = -\frac{\ln x}{x}$ by FTC, Part 1.

5. Below is the graph of $f(x)$.



(a) (4 pts) Find $\int_{-3}^{5} f(x)\,dx$.

$$\int_{-3}^{5} f(x)dx = \frac{1^2 \cdot \pi}{2} - \frac{1}{2}\cdot 1\cdot 2 - 1\cdot 1 - \frac{1}{2}\cdot 1\cdot 1 - 1\cdot 1 - \frac{1}{2}\cdot 1\cdot 1 + \frac{1}{2}\cdot 2\cdot 1 + 2\cdot 1$$
$$= \frac{\pi}{2} - 4 + 3 = \boxed{\frac{\pi}{2} - 1}$$

(b) (4 pts) Find $\int_{-3}^{5} (2f(x) + 1)\,dx$.

$$\int_{-3}^{5} (2f(x)+1)dx = 2\int_{-3}^{5} f(x)dx + 1(5-(-3))$$
$$= 2\left(\frac{\pi}{2} - 1\right) + 8 = \pi - 2 + 8 = \boxed{\pi + 6}$$

(c) (5 pts) Find $\int_{-3}^{3} 4f(x)\,dx + \int_{3}^{5} (2f(x) - 3)\,dx$.

$$\int_{-3}^{3} f(x)dx = \frac{\pi}{2} - 4 \qquad \int_{3}^{5} f(x)dx = 3$$

$$\int_{-3}^{3} 4f(x)dx + \int_{3}^{5} (2f(x)-3)dx = 4\left(\frac{\pi}{2} - 4\right) + 2(3) - 3(5-3)$$
$$= 2\pi - 16 + 6 - 6 = \boxed{2\pi - 16}$$

6. Evaluate the following integrals:

(a) (6 pts) $\int_1^2 3x^2 + 2\,dx$ $= \frac{3x^3}{3} + 2x \Big|_1^2 = x^3 + 2x \Big|_1^2 = \left((2)^3 + 2\cdot 2\right) - \left(1^3 + 2\cdot 1\right)$

$= (8+4) - (3)$

$= \boxed{9}$

(b) (6 pts) $\int \left[x^2 + \sqrt{x} + 1 + \frac{1}{x} + \frac{1}{x^2+1}\right] dx$

$= \boxed{\frac{x^3}{3} + \frac{2}{3}x^{3/2} + x + \ln x + \tan^{-1} x + C}$

(c) (6 pts) $\int \frac{3x^4 + x^3 e^{x^2}}{x^2}\, dx$

$$= \int 3x^2 + xe^{x^2}\,dx = \int 3x^2\,dx + \overbrace{\int xe^{x^2}\,dx}^{u\text{-sub}} \qquad \begin{array}{l} u = x^2 \\ du = 2x\,dx \end{array}$$

$$= x^3 + \int e^u \frac{du}{2} = x^3 + \frac{e^u}{2} + C$$

$$= \boxed{x^3 + \frac{e^{x^2}}{2} + C}$$

(d) (6 pts) $\int_0^1 \sec^2(x) \cdot \cos(\tan(x))\, dx$ u-sub

$$u = \tan x$$
$$du = \sec^2 x\,dx$$

$$\int_0^1 \sec^2 x \cdot \cos(\tan x)\,dx = \int_{x=0}^{x=1} \cos(u)\,du = \sin(u)\Big|_{x=0}^{x=1} = \sin(\tan x)\Big|_0^1$$

$$= \sin(\tan(1)) - \sin(\tan(0))$$

$$= \boxed{\sin(\tan(1))}$$

(e) (6 pts) $\int \sin x + e^x \, dx$

$$= -\cos x + e^x + C$$

(f) (6 pts) $\int x^2(x^3+2)^5 \, dx$

u-sub is easiest

$$u = x^3 + 2$$

$$du = 3x^2 \, dx$$

$$\int x^2(x^3+2)^5 dx = \int \frac{1}{3} u^5 \, du = \frac{u^6}{18} + C = \frac{(x^3+2)^6}{18} + C$$

7. A particle moves in a straight line with the given acceleration function and velocity at $t = 1$

$$a(t) = 3t^2 - 12t + 9 \qquad v(1) = 4$$

(a) (4 pts) Find an equation for the velocity of the particle at time t.

$$v(t) = \int a(t)\, dt = \int 3t^2 - 12t + 9\, dt$$

$$= t^3 - 6t^2 + 9t + C$$

$$4 = v(1) = 1 - 6 + 9 + C = 4 + C \Rightarrow C = 0$$

$$\boxed{v(t) = t^3 - 6t^2 + 9t}$$

(b) (4 pts) Find the displacement of the particle on the interval $-2 \le t \le 4$.

$$\int_{-2}^{4} t^3 - 6t^2 + 9t\, dt = \frac{t^4}{4} - 2t^3 + \frac{9}{2}t^2 \Big|_{-2}^{4}$$

$$= 4^3 - 2 \cdot 4^3 + 9(8) - (4 + 16 + 18)$$

$$= \boxed{-30}$$

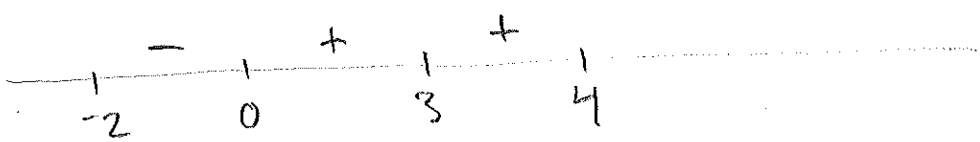
(c) (8 pts) Find the total distance traveled by the particle on the interval $-2 \leq t \leq 4$.

(Hint: $v(t)$ should be easy to factor.)

need to solve $v(t) = 0$

$$t^3 - 6t^2 + 9t = 0$$

$$t(t-3)^2 = 0 \qquad \text{i.e. } t = 0, 3$$



$v(-1) = -16 \qquad v(1) = 4 \qquad v(4) = 4$

$$\text{total distance} = \int_{-2}^{4} |v(t)|\,dt = \int_{-2}^{0} -v(t)\,dt + \int_{0}^{4} v(t)\,dt$$

$$= \frac{t^4}{4} - 2t^3 + \frac{9}{2}t^2 \Big|_0^4 - \frac{t^4}{4} - 2t^3 + \frac{9}{2}t^2 \Big|_{-2}^{0}$$

$$= 8 + 38 = \boxed{46}$$