Homework 7 - Key

7.8 - 3, 7, 8, 13, 20, 67

3. $\int_1^t \frac{1}{x^3}\,dx = -\frac{1}{2}\cdot\frac{1}{x^2}\Big|_1^t = -\frac{1}{2t^2} + \frac{1}{2} = g(t)$

$g(10) = \frac{1}{2} - \frac{1}{2\cdot(10)^2} = 99/200 \approx .495$

$g(100) = \frac{1}{2} - \frac{1}{2\cdot(100)^2} \approx .49995$

$g(1000) = \frac{1}{2} - \frac{1}{2\cdot(1000)^2} \approx .4999995$

total area: $\lim_{t\to\infty} \int_1^t \frac{1}{x^3}\,dx = \lim_{t\to\infty} \frac{1}{2} - \frac{1}{2t^2} = \boxed{\frac{1}{2}}$

7. $\int_{-\infty}^0 \frac{1}{3-4x}\,dx = \lim_{t\to-\infty} \int_t^0 \frac{1}{3-4x}\,dx =$ $u = 3-4x$, $du = -4\,dx$

$= \lim_{t\to-\infty} \int_t^0 \left(-\frac{1}{4}\right)\frac{1}{u}\,du = \lim_{t\to-\infty} \left(-\frac{1}{4}\right)\ln(u)\Big|_t^0$

$= \lim_{t\to-\infty} \left(-\frac{1}{4}\right)\ln(3-4x)\Big|_t^0$

$= \lim_{t\to-\infty} -\frac{1}{4}\ln 3 + \underbrace{\frac{1}{4}\ln(3-4t)}_{\to\infty}$

$\boxed{\text{DIVERGES}}$

$\longrightarrow \infty$

8. $$\int_1^\infty \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{(2x+1)^3}\,dx$$

$$= \lim_{t\to\infty} \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{(2x+1)^2}\right)\Big|_1^t$$

$$= \lim_{t\to\infty} -\frac{1}{4(2x+1)^2}\Big|_1^t$$

$$= \lim_{t\to\infty} \underbrace{-\frac{1}{4(2t+1)^2}}_{\to 0} + \frac{1}{4\cdot 9} = \boxed{\frac{1}{36}}$$

CONVERGENT

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13. $$\int_{-\infty}^{\infty} xe^{-x^2}\,dx = \int_0^\infty xe^{-x^2}\,dx + \int_{-\infty}^0 xe^{-x^2}\,dx$$

$$\int_0^\infty xe^{-x^2}\,dx = \lim_{t\to\infty}\int_0^t xe^{-x^2}\,dx \qquad \begin{matrix} u=-x^2 \\ du=-2x\,dx \end{matrix}$$

$$= \lim_{t\to\infty}\int_0^t \left(-\frac{1}{2}\right)e^u\,du = \lim_{t\to\infty} \left(-\frac{1}{2}\right)e^u\Big|_0^t$$

$$= \lim_{t\to\infty} \left(-\frac{1}{2}\right)e^{-x^2}\Big|_0^t$$

$$= \lim_{t\to\infty} \underbrace{\left(-\frac{1}{2}\right)e^{-t^2}}_{\to 0} + \frac{1}{2} = \frac{1}{2}$$

$$\int_{-\infty}^0 xe^{-x^2}\,dx = \lim_{t\to-\infty}\int_t^0 xe^{-x^2}\,dx = \lim_{t\to-\infty} -\frac{e^{-x^2}}{2}\Big|_t^0$$

$$= \lim_{t\to-\infty} -\frac{1}{2} + \underbrace{\frac{e^{-t^2}}{2}}_{\to 0} = -\frac{1}{2}$$

$$\frac{1}{2} + -\frac{1}{2} = \boxed{0}$$

20. $\int_2^\infty ye^{-3y}\,dy = \lim_{t\to\infty}\int_2^t ye^{-3y}\,dy$

int by parts
$u=y \quad dv=e^{-3y}dy$
$du=dy \quad v=-\frac{1}{3}e^{-3y}$

$= \lim_{t\to\infty}\left(-\frac{ye^{-3y}}{3}\Big|_2^t + \int_2^t \frac{e^{-3y}}{3}dy\right)$

$= \lim_{t\to\infty}\left(-\frac{ye^{-3y}}{3} - \frac{e^{-3y}}{9}\Big|_2^t\right)$

$= \lim_{t\to\infty}\left(\underbrace{-\frac{te^{-3t}}{3}}_{\text{Need L'hospital}} - \underbrace{\frac{e^{-3t}}{9}}_{\to 0} + \frac{2\cdot e^{-6}}{3} + \frac{e^{-6}}{9}\right)$

$\lim_{t\to\infty}\frac{-te^{-3t}}{3} = \lim_{t\to\infty}\frac{-t}{3e^{3t}} \overset{\boxed{L}}{=} \lim_{t\to\infty}\frac{-1}{9e^{3t}} = 0$

$= \frac{2e^{-6}}{3} + \frac{e^{-6}}{9} = \frac{7e^{-6}}{9}$

$\approx .001928$

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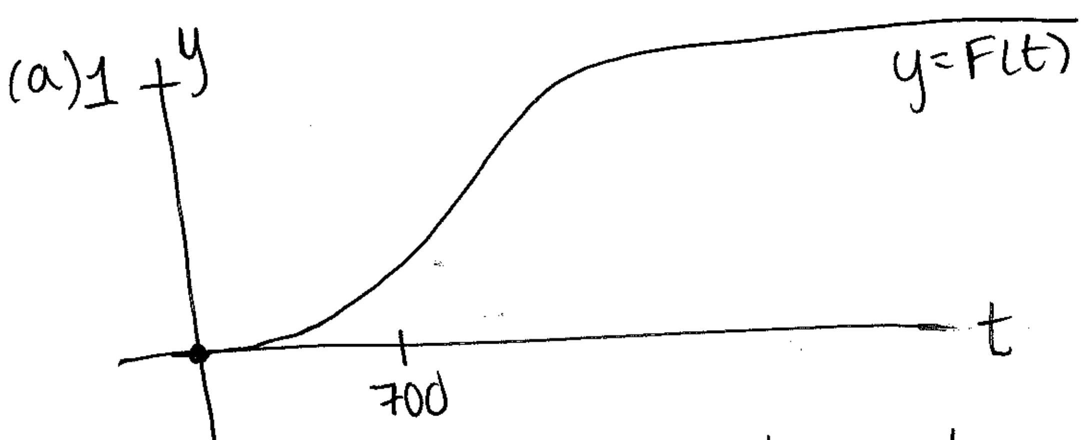
67. $F(t) = \frac{\text{\# of bulbs that are burned out at time } t}{\text{total \# bulbs}}$

(a)



(b) The rate at which lightbulbs burn out

(c) $\int_0^\infty r(t)dt = \lim_{t\to\infty}\int_0^t r(t)dt = \lim_{t\to\infty}\left(F(t)-F(0)\right) = \boxed{1}$

as $t\to\infty$, $F(t)\to 1$ b/c all lightbulbs burn out eventually.