HW3 5.5: 6, 24, 32, 62, 70, 77

6.1: 2, 4, 14, 20, 47ab

5.5

6. $u = 1/x$

$du = -1/x^2\,dx$

$$\int \frac{\sec^2(1/x)}{x^2}\,dx = \int -\sec^2 u\,du = -\tan u + C = -\tan \frac{1}{x} + C$$

24. $$\int \sqrt{x}\sin(1+x^{3/2})\,dx$$

$u = 1 + x^{3/2}$

$du = \frac{3}{2}x^{1/2}\,dx = 2\sqrt{x}\,dx$

$$= \int \frac{2}{3}\sin u\,du$$

$$= -\frac{2}{3}\cos u + C = -\frac{2}{3}\cos(1+x^{3/2}) + C$$

32. $$\int \frac{\sin(\ln x)}{x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \int \sin u\,du$$

$$= -\cos u + C = -\cos(\ln x) + C$$

62. $$\int_0^{\pi/2} \cos x \sin(\sin x)\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$= \int_{x=0}^{x=\pi/2} \sin u\,du$$

$$= -\cos u \Big|_{x=0}^{x=\pi/2} = -\cos(\sin x)\Big|_0^{\pi/2} = -\cos(\sin \tfrac{\pi}{2}) + \cos(\sin 0)$$

$$= -\cos(1) + \cos(0)$$

$$= 1 - \cos(1)$$

$$= .4597$$

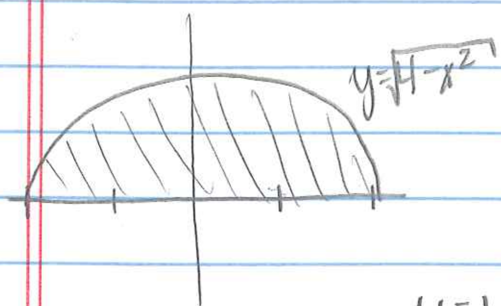
70. $$\int_0^{1/2} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx \qquad u = \sin^{-1}x \qquad du = 1/\sqrt{1-x^2}\,dx$$

$$= \int_{x=0}^{x=1/2} u\,du = \frac{u^2}{2}\Big|_{x=0}^{x=1/2} = \frac{(\sin^{-1}x)^2}{2} = \frac{(\sin^{-1}(1/2))^2 - (\sin^{-1}0)^2}{2}$$

$$= \frac{(\pi/6)^2 - 0}{2} = \boxed{\pi^2/72}$$

$$= .13708$$

77. $$\int_{-2}^{2} (x+3)\sqrt{4-x^2}\,dx = \int_{-2}^{2} x\sqrt{4-x^2}\,dx + \int_{-2}^{2} 3\sqrt{4-x^2}\,dx$$



$$3\int_{-2}^{2}\sqrt{4-x^2}\,dx = 3\left(\frac{\pi 2^2}{2}\right) = 6\pi$$

$$u = 4-x^2$$
$$du = -2x\,dx$$

$$\int_{-2}^{2} x\sqrt{4-x^2}\,dx = \int_{x=-2}^{x=2} -\frac{1}{2}\sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big|_{x=-2}^{x=2}$$

$$= -\frac{1}{3}(4-x^2)^{3/2}\Big|_{-2}^{2}$$

$$= -\frac{1}{3}(4-4)^{3/2} - \left(-\frac{1}{3}(4-4)\right)^{3/2}$$

$$= 0$$

$$\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx = 6\pi + 0 = \boxed{6\pi}$$

6.1

2. $\text{Area} = \int_0^2 \sqrt{x+2} - \frac{1}{x+1}\, dx = \frac{2}{3}(x+2)^{3/2} - \ln(x+1) \Big|_0^2$

$= \frac{2}{3}(2+2)^{3/2} - \ln(2+1) - \frac{2}{3}(0+2)^{3/2} + \ln(0+1)$

$= \frac{2}{3} \cdot 8 - \ln 3 - \frac{2}{3} 2\sqrt{2} + 0 = \frac{16}{3} - \ln 3 - \frac{4\sqrt{2}}{3} = 2.3491$

4. $\int_0^3 (2y-y^2)-(y^2-4y)\, dy = \int_0^3 6y - 2y^2 dy = 3y^2 - \frac{2}{3}y^3 \Big|_0^3$

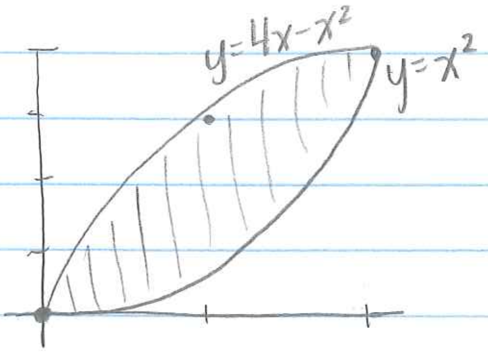
$= 3(3)^2 - \frac{2}{3}(3)^3 = 27 - 18 = 9$

14. Intersection Points: $x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$x(2x-4) = 0$

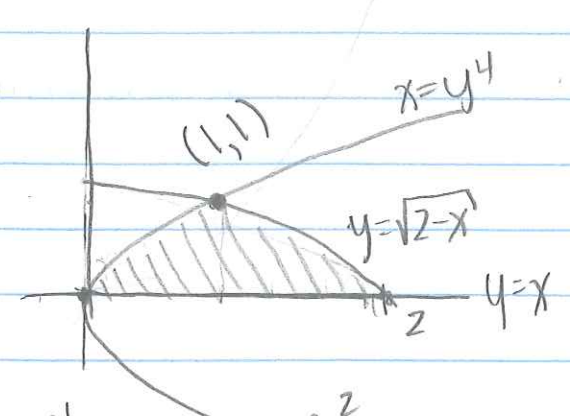
$x = 0$ OR $x = 2$



$\text{Area} = \int_0^2 4x - x^2 - x^2\, dx = \int_0^2 4x - 2x^2 dx = 2x^2 - \frac{2}{3}x^3 \Big|_0^2$

$= 8 - \frac{16}{3} = \boxed{\frac{8}{3}} = 2.66667$

20.



Intersection points

$y^4 = 2 - y^2$

$y^4 + y^2 - 2 = 0$

$(y^2+2)(y^2-1) = 0$

$\cancel{y^2 = -2}$ OR $y^2 = 1$

$y = \pm 1$

$\text{Area} = \int_0^1 x^{1/4} dx + \int_1^2 \sqrt{2-x}\, dx = \frac{4}{5}x^{5/4}\Big|_0^1 + -\frac{2}{3}(2-x)^{3/2}\Big|_1^2$

$= \frac{4}{5} + \frac{2}{3} = \boxed{22/15} = 1.46667$

47. (a) Car A; it has had a higher velocity for the entire first minute (the curve for A is higher than the curve for B)

(b) The area of the shaded region is the distance between the two cars after 1 minute.