Final Review

Studying for Your Final Exam

- Study your previous exams. (Answer keys are posted on the "Exams" page of the website). If you can't do every problem on them, you're not ready for your final.
- Study your previous homeworks and quizzes (Answer keys are posted in the "Homework" and "Exam" pages of the website).
- Use this review sheet and the review sheets given out for the other midterms. For problems you've already done on the other reviews, solve them again without looking at your solution.
- Go to tutorials or office hours to ask questions about topics you don't understand well.

Section 7.4 - Partial Fraction Decomposition

Evaluate the following integrals using partial fraction decomposition

1. $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}\,dx$
2. $\displaystyle\int \frac{2}{(2x+1)(x+1)}\,dx$
3. $\displaystyle\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$
4. $\displaystyle\int \frac{2x^2-10x+32}{(2x+1)(x-2)^2}\,dx$
5. $\displaystyle\int \frac{x^3+3x^2-4x-4}{x^4-4x^2}\,dx$

7.8 - Improper Integrals

Determine if the following improper integrals converge or diverge. If an integral converges, figure out what it converges to.

1. $\displaystyle\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{x^2+x}\,dx$
3. $\displaystyle\int_{-\infty}^\infty \cos \pi t\,dt$
4. $\displaystyle\int_2^\infty \frac{dv}{v^2+2v-3}$
5. $\displaystyle\int_{-\infty}^\infty xe^{-2x^2}\,dx$
6. $\displaystyle\int_1^\infty \frac{1}{2x+3}\,dx$

8.1 - Arc Length

1. Find the length of the curve $y = 1 + 2x^{3/2}$ for $0 \le x \le 1$
2. Find the length of the curve $y^2 = 2(x+3)^3$ for $0 \le x \le 2$ and $y > 0$
3. Find the length of the curve $y = \ln(\sec x)$ for $0 \le x \le \pi/4$ (Use the integral table in the back of your book)

4. Show, using the arc length formula, that the circumference of a circle of radius r is $2\pi r$.

8.2 - Surface Area

1. Find the surface area of the shape created by rotating $y = x^3$ from $0 \leq x \leq 2$ about the x-axis.
2. Find the surface area of the shape created by rotating $3x = y^2 + 6$ from $2 \leq y \leq 6$ about the x-axis.
3. Find the surface area of the shape created by rotating $x = \frac{1}{3}(y^2+2)^{3/2}$ from $1 \leq y \leq 2$ about the x-axis.
4. Find the surface area of the shape created by rotating $y = 2 - x^2$ from $0 \leq x \leq 1$ about the y-axis.
5. Find the surface area of the shape created by rotating $x = \sqrt{a^2 - y^2}$ from $0 \leq y \leq a/2$ about the y-axis, where a is any real number.
6. Show that the surface area of a sphere of radius r is equal to $4\pi r^2$

Integrals

First decide what integration method you need to evaluate each of the following integrals, then do it.

1. $\displaystyle\int_1^3 \frac{\ln(2x)}{x^2}\,dx$
2. $\displaystyle\int \frac{2}{2x^2+3x+1}\,dx$
3. $\displaystyle\int e^{-2t}\cdot\cos t\,dt$
4. $\displaystyle\int x^2 e^{4x}\,dx$
5. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$
6. $\displaystyle\int \tan^4(s)\sec^4(s)\,ds$
7. $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$
8. $\displaystyle\int \sin^3 x\cos^2 x\,dx$
9. $\displaystyle\int x^3\sqrt{4-x^2}\,dx$
10. $\displaystyle\int \frac{x}{\sqrt{x^2-1}}\,dx$
11. $\displaystyle\int \frac{\cos(\ln x)\sin(\ln x)}{x}\,dx$
12. $\displaystyle\int \arcsin(x)\,dx$
13. $\displaystyle\int_2^4 \frac{(2x+1)}{x^3+x^2}\,dx$
14. $\displaystyle\int \frac{dx}{(x+2)(x-5)}$
15. $\displaystyle\int x^2\cos x\,dx$
16. $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

Answers

- 7.4
  1. $\frac{\ln(2x+1)}{2} + 2\ln(x-1) + C$
  2. $2\ln(2x+1) - 2\ln(x+1) + C$
  3. 1.76549
  4. $\frac{11}{5}\ln(2x+1) - \frac{6}{5}\ln(x-2) - \frac{12}{x-2} + C$
  5. $\ln(x) - \frac{4}{x} + \ln(x+1) - \ln(x-1) + C$
- 7.8
  1. 2
  2. $\ln(2)$
  3. Diverges
  4. $\frac{\ln(5)}{4}$
  5. 0
  6. Diverges
- 8.1
  1. $\frac{20\sqrt{10}}{27} - \frac{2}{27} = 2.26835$
  2. 8.6972
  3. $\ln(\sqrt{2}+1) = .881374$
  4. $2\pi r$
- 8.2
  1. $\frac{(149\cdot\sqrt{149}-1)\cdot\pi}{27} = 203.044$
  2. $\frac{(577\cdot\sqrt{577}-65\cdot\sqrt{65})\cdot\pi}{24} = 1745.67$
  3. $\frac{21\pi}{2}$
  4. $\frac{(5\cdot\sqrt{5}-1)\cdot\pi}{6} = 5.33041$
  5. $\pi \cdot a^2$
  6. $4\pi r^2$
- Integrals
  1. Integration by Parts; $\frac{\ln 2}{2} + \frac{2}{3} - \frac{\ln 6}{2}$
  2. Partial Fraction Decomposition; $2\ln(2x+1) - 2\ln(x+1) + C$
  3. Integration by Parts; $\frac{1}{5}(e^{-2t}\cdot\sin t - 2e^{-2t}\cdot\cos t) + C$
  4. Integration by Parts; $\frac{x^2e^{4x}}{4} - \frac{xe^{4x}}{8} - \frac{e^{4x}}{32} + C$

5. U-Substitution; $-2\cos\sqrt{x}+C$
6. Trig Integral Techniques; $\frac{\tan^7 s}{7}+\frac{\tan^5 s}{5}+C$
7. Partial Fraction Decomposition; $\ln(x)-\ln(x+1)+\ln(x-1)+C$
8. Trig Integral Techniques; $\frac{\cos^5 x}{5}-\frac{\cos^3 x}{3}+C$
9. Trig Substitution and Trig Integral Techniques; $\frac{(4-x^2)^{5/2}}{5}-\frac{4(4-x^2)^{3/2}}{3}+C$
10. U-Substitution or Trig Sub (Harder); $\sqrt{x^2-1}+C$
11. U-Substitution Twice; $\frac{\sin^2(\ln x)}{2}+C$
12. Integration by Parts; $x\arcsin x+\sqrt{1-x^2}+C$
13. Partial Fraction Decomposition; $\ln(6/5)+1/4$
14. Partial Fraction Decomposition; $\frac{1}{7}\ln(x-5)-\frac{1}{7}\ln(x+2)+C$
15. Integration by Parts; $x^2\sin x+2x\cos x-2\sin x+C$
16. U-Substitution; $-\arctan(cosx)+C$