

Quiz 5: Partial Fractions

February 27, 2013

Name: Key Section: Adelstem

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Evaluate $\int \frac{3x}{x^2+x-2} dx = \int \frac{3x}{(x-1)(x+2)} dx$

$$\frac{3x}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$

$$3x = A(x+2) + B(x-1)$$

$$3x = (A+B)x + (2A-B) \cdot 1$$

$$A+B=3 \quad 2A-B=0$$

$$A+2A=3$$

$$2A=B$$

$$3A=3$$

$$B=2$$

$$A=1$$

$$\int \frac{3x}{x^2+x-2} dx = \int \frac{1}{x-1} + \frac{2}{x+2} dx = \ln|x-1| + 2 \ln|x+2| + C$$

2. Evaluate $\int \frac{x^2+8}{x^3-4x^2+4x} dx = \int \frac{x^2+8}{x(x-2)^2} dx$

$$\frac{x^2+8}{x^3-4x^2+4x} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$x^2+8 = A(x^2-4x+4) + B \cdot x(x-2) + C \cdot x$$

$$x^2+8 = (A+B)x^2 + (-4A-2B+C)x + 4A$$

$$4A=8 \quad A+B=1 \quad -4A-2B+C=0$$

$$A=2 \quad B=-1 \quad -8+2+C=0$$

$$C=6$$

$$\int \frac{x^2+8}{x^3-4x^2+4x} dx = \int \frac{2}{x} - \frac{1}{x-2} + \frac{6}{(x-2)^2} dx$$

$$= 2 \ln(x) - \ln(x-2) - \frac{6}{x-2} + C$$