Quiz 4: Avgerage Value and Integration by Parts

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Name: Key Section: Abel-Stem

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Find the average value of the function $f(x) = x \cdot \cos x$ on the interval $[0, \pi]$.

$$\text{Avg value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\text{Avg value} = \frac{1}{\pi - 0}\int_0^{\pi} x\cdot\cos x\,dx \qquad u = x \qquad dv = \cos x\,dx$$

$$du = dx \qquad v = \sin x$$

$$= \frac{1}{\pi}\left(x\cdot\sin x - \int_0^{\pi}\sin x\,dx\right) = \frac{1}{\pi}\left(x\sin x + \cos x\right)\Big|_0^{\pi}$$

$$= \frac{1}{\pi}\left(\pi\sin\pi + \cos\pi\right) - \frac{1}{\pi}\left(0 + \cos 0\right)$$

$$= \frac{1}{\pi}(0-1) - \frac{1}{\pi}(1) = \boxed{-\frac{2}{\pi}}$$

note: $\frac{x^2}{2}\cdot\sin x$ is NOT an antiderivative of $x\cdot\cos x$

2. Evaluate $\int x^2 \ln x \, dx$.

$u = \ln x$ $\qquad dv = x^2 \, dx$

$du = \frac{1}{x} dx$ $\qquad v = \frac{x^3}{3}$

$$\int x^2 \ln x \, dx = \frac{x^3}{3} \cdot \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 \, dx$$

$$= \boxed{\frac{x^3}{3} \cdot \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C}$$