# Chapter 6 Review

February 6, 2012

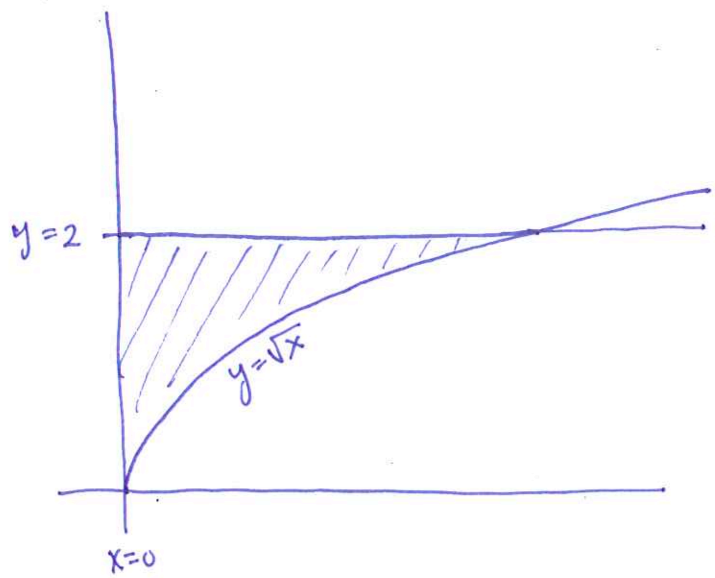
Name: ______________________ Section: ____________________

Instructions: This worksheet is intended to guide you through the steps to finding areas between curves and computing volumes of solids of revolution using slices and shells.

For the following problems, consider the region enclosed by the curves

$$x = 0 \qquad y = 2 \qquad y = \sqrt{x}.$$

1. Sketch the region:



2. Finding area of this region.

(a) Find the area of this region with respect to $x$.

- What is the general formula for area between two curves? (should have $dx$)

$$A = \int_a^b f(x) - g(x)\,dx$$

- $a =$ 0
- $b =$ 4
- $f(x) =$ 2
- $g(x) =$ $\sqrt{x}$
- And so area is:

$$A = \int_0^4 2 - \sqrt{x}\,dx = 2x - \frac{2}{3}x^{3/2}\Big|_0^4 = 2(4) - \frac{2}{3}(4)^{3/2} - \left(2(0) - \frac{2}{3}(0)^{3/2}\right)$$

$$= 8 - \frac{2}{3}(8) = \frac{8}{3}.$$

(b) Find the area of this region with respect to $y$.

- What is the general formula for area between two curves? (should have $dy$)

$$A = \int_c^d f(y) - g(y)\,dy$$

- $c =$ 0
- $d =$ 2
- $f(y) =$ $\cancel{2}\ y^2$
- $g(y) =$ 0
- And so area is:

$$A = \int_0^2 y^2 - 0\,dy = \frac{y^3}{3}\Big|_0^2 = \frac{2^3}{3} - \frac{0}{3} = \frac{8}{3}.$$

3. Finding volume of the solid of revolution obtained by rotating this region about the $x$-axis.

(a) Find the volume using slices.

- Are the slices horizontal or vertical? vertical
- So will our integral have $dx$ or $dy$? $dx$
- What is the general equation of volume (using slices)?

$$V = \int_a^b A(x)\,dx$$

- Are our slices disks or washers? washers
- $A(x) = \pi R^2 - \pi r^2 = \pi f(x)^2 - \pi g(x)^2 = \pi (2)^2 - \pi (\sqrt{x})^2 = 4\pi - \pi x$
- $a =$ $0$
- $b =$ $4$
- And so the volume is

$$V = \int_0^4 4\pi - \pi x\,dx = 4\pi x - \frac{\pi x^2}{2}\Big|_0^4 = 4\pi(4) - \frac{\pi(4)^2}{2} - \left(4\pi(0) - \frac{\pi(0)^2}{2}\right)$$

$$= 16\pi - \frac{16\pi}{2} = 8\pi.$$

(b) Find the volume using shells.

- Will our integral have $dx$ or $dy$? $dy$
- What is the general equation of volume (using shells)?

$$V = \int_c^d 2\pi y\, h(y)\,dy$$

- $h(y) =$ $f(y) - g(y) = y^2 - 0 = y^2$
- $c =$ $0$
- $d =$ $2$
- So the volume is

$$V = \int_0^2 2\pi y(y^2)\,dy = \int_0^2 2\pi y^3\,dy = \frac{2\pi y^4}{4}\Big|_0^2$$

$$= \frac{2\pi(2)^4}{4} - \frac{2\pi(0)^4}{4} = 8\pi.$$

4. Finding volume of the solid of revolution obtained by rotating this region about the $y$-axis.

(a) Find the volume using slices.

- Are the slices horizontal or vertical? horizontal
- So will our integral have $dx$ or $dy$? $dy$
- What is the general equation of volume (using slices)?

$$V = \int_c^d A(y)\,dy$$

- Are our slices disks or washers? discs

- $A(y) = \pi r^2 = \pi f(y)^2 = \pi (y^2)^2 = \pi y^4$

- $c =$ $0$

- $d =$ $2$

- And so the volume is

$$V = \int_0^2 \pi y^4\,dy = \frac{\pi y^5}{5}\Big|_0^2 = \frac{\pi (2)^5}{5} - \frac{\pi (0)^5}{5} = \frac{32\pi}{5}$$

(b) Find the volume using shells.

- Will our integral have $dx$ or $dy$? $dx$
- What is the general equation of volume (using shells)?

$$V = \int_a^b 2\pi x\, h(x)\,dx$$

- $h(x) =$ $f(x) - g(x) = 2 - \sqrt{x}$

- $a =$ $0$

- $b =$ $4$

- So the volume is

$$V = \int_0^4 2\pi x(2-\sqrt{x})\,dx = \int_0^4 4\pi x - 2\pi x^{3/2}\,dx = \frac{4\pi x^2}{2} - \frac{4\pi x^{5/2}}{5}\Big|_0^4$$



$$= 2\pi(4)^2 - \frac{4\pi}{5}(4)^{5/2} - \left(2\pi(0)^2 - \frac{4\pi}{5}(0)^{5/2}\right) = 32\pi - \frac{4\pi}{5}(32) = \frac{32\pi}{5}.$$