

Quiz 7: Trig Integrals and Trig Substitution

February 22, 2012

Name: Solutions Section: _____

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. This quiz has two sides.

1. Find $\int \sin^5(x) \cos^3(x) dx$.

Method # 1 :

$$\int \sin^5 x \cos^3 x dx = \int \sin^4 x \cos^2 x \cos x dx = \int \sin^4 x (1 - \sin^2 x) \cos x dx$$

Let $u = \sin x$. $du = \cos x dx$. The above becomes

$$\int u^4 (1 - u^2) du = \int u^4 - u^6 du = \frac{u^5}{5} - \frac{u^7}{7} + C = \boxed{\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C}$$

Method # 2 :

$$\int \sin^5 x \cos^3 x dx = \int \sin^4 x \cos^2 x \sin x dx = \int (1 - \cos^2 x)^2 \cos^2 x \sin x dx$$

Let $u = \cos x$ $du = -\sin x dx$. The above becomes

$$-\int (1 - u^2)^2 u^2 du = -\int (1 - 2u^2 + u^4) u^2 du = -\int u^2 - 2u^4 + u^6 du = -\frac{u^3}{3} + \frac{2u^5}{5} - \frac{u^7}{7} + C$$

$$= \boxed{-\frac{\cos^3 x}{3} + \frac{2\cos^5 x}{5} - \frac{\cos^7 x}{7} + C}$$

2. Solve the following integral using trig substitution: $\int \frac{x^3}{\sqrt{1-x^2}} dx$.

Let $x = \sin \theta$. Then $dx = \cos \theta d\theta$.

$$\int \frac{x^3}{\sqrt{1-x^2}} dx = \int \frac{\sin^3 \theta}{\cos \theta} \cos \theta d\theta = \int \sin^3 \theta d\theta = \int (1 - \cos^2 \theta) \sin \theta d\theta$$

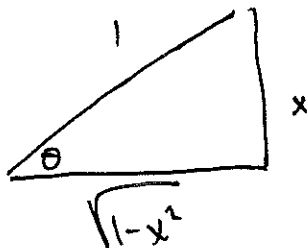
Let $u = \cos \theta$. Then $du = -\sin \theta d\theta$. The above becomes

$$-\int (1-u^2) du = \int u^2 - 1 du = \frac{u^3}{3} - u + C = \frac{\cos^3 \theta}{3} - \cos \theta + C.$$

To figure out what $\cos \theta$ is in terms of x , we need to use the triangle.

$x = \sin \theta = \frac{\text{opp}}{\text{hyp}}$. Let $\text{opp} = x$ and $\text{hyp} = 1$.

By Pythagorean Thm,
adj = $\sqrt{1-x^2}$



$$\therefore \cos \theta = \sqrt{1-x^2}.$$

$$\text{Soln: } \boxed{\frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2} + C}$$

Extra Credit: Find $\int_{-\pi}^{\pi} \underbrace{\cos^{100}(x) \sin^{99}(x)}_{\text{odd}} dx$.

$$= \boxed{0}$$

$\cos(x)$ even $\Rightarrow \cos^{100}(x)$ even

$\sin(x)$ odd $\Rightarrow \sin^{99}(x)$ odd

even times odd is odd.

Surprise Quiz

February 23, 2012

Name: Solutions. Section: _____

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. This quiz has two sides.

1. Solve the following integral using trig substitution: $\int \frac{x}{\sqrt{x^2-1}} dx$.

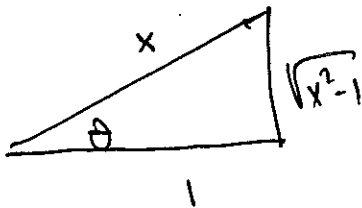
Trig-Sub: let $x = \sec \theta$.

$$dx = \sec \theta \tan \theta d\theta$$

$$\int \frac{x}{\sqrt{x^2-1}} dx = \int \frac{\sec \theta}{\tan \theta} \sec \theta \tan \theta d\theta = \int \sec^2 \theta d\theta = \tan \theta + C.$$

Triangle: $x = \sec \theta = \frac{\text{hyp}}{\text{adj}}$ let hyp = x and adj = 1.

By Pythagorean thm, opp = $\sqrt{x^2-1}$



So $\tan \theta = \sqrt{x^2-1}$.

Soln: $\boxed{\sqrt{x^2-1} + C}$