Quiz 2: Antiderivatives and Areas

January 18, 2012

Name: ______________________ Section: ____________________

Instructions: Be sure to write neatly and show all steps. Circle or box your final answer. Answer both questions (second one is on the back).

1. Given $f''(x) = 15\sqrt{x} - 9\sin(3x)$ with $f'(0) = 4$ and $f(0) = -1$, find $f(x)$.

$$f''(x) = 15x^{1/2} - 9\sin(3x)$$

$$f'(x) = 15 \cdot \frac{1}{3/2} x^{3/2} + 9 \cdot \left(\tfrac{1}{3}\right)\cos(3x) + c$$

$$= 15 \cdot \left(\tfrac{2}{3}\right) x^{3/2} + 3\cos(3x) + c$$

$$= 10x^{3/2} + 3\cos(3x) + c$$

since $f'(0)=4$,

$$10(0)^{3/2} + 3\cos(3 \cdot 0) + c = 4$$

$$0 + 3 + c = 4$$

$$\boxed{c = 1}$$

so $\boxed{f'(x) = 10x^{3/2} + 3\cos(3x) + 1}$

$$f(x) = \frac{10}{5/2} x^{5/2} + 3\left(\tfrac{1}{3}\right)\sin(3x) + x + c$$

$$= 10\left(\tfrac{2}{5}\right)x^{5/2} + 1 \cdot \sin(3x) + x + c$$

$$= 4x^{5/2} + \sin(3x) + x + c$$

$$f(0) = 4(0)^{5/2} + \sin(3 \cdot 0) + 0 + c = -1$$

$$0 + 0 + 0 + c = -1$$

$$\boxed{c = -1}$$

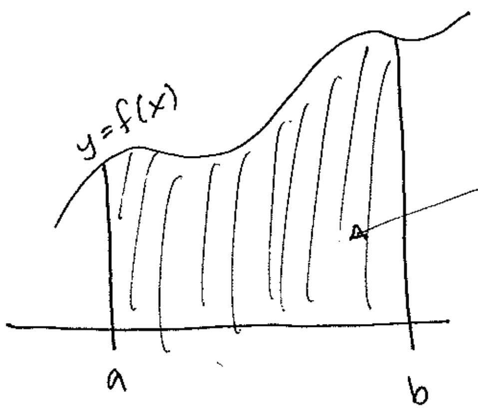
$$f(x) = 4x^{5/2} + \sin(3x) + x - 1$$

1

2. In class, we found that the area under the curve $y = f(x)$ between a and b is

$$A = \lim_{n\to\infty} \Big(f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_n)\Delta x \Big).$$

Explain in your own words where this equation comes from. Be sure to be specific and explain the meaning of each symbol.

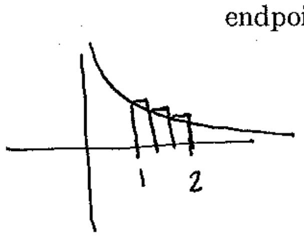


In order to find the area of this region under the curve $y=f(x)$, we divide the region into n strips and approximate the area of each strip with a rectangle. Then we add the areas of these rectangles to get an approximation for the area of the entire region. There will be n rectangles since there are n strips. The width of each rectangle is $\Delta x = \frac{b-a}{n}$ and the height of the i^{th} rectangle is $f(x_i)$ where x_i is a sample point (such as the right endpoint) from the i^{th} subinterval. Therefore the area of the i^{th} rectangle is $f(x_i)\Delta x$. For example, the area of the 1^{st} rectangle is $f(x_1)\Delta x$, 2^{nd} rectangle is $f(x_2)\Delta x$, and so on. For larger and larger n, our approximation gets better and better. That is, we get a better approximation of area as limit $\to \infty$, because we have more rectangles. So we define area as this limit.

Extra Credit: Circle one to complete the statement:

For the function $f(x) = 1/x$, if we approximate the area between $x = 1$ and $x = 2$ with left endpoint Riemann sums, our approximation is an:

over-approximation   under-approximation



overapproximation since $f(x)$ is decreasing on $[1,2]$.