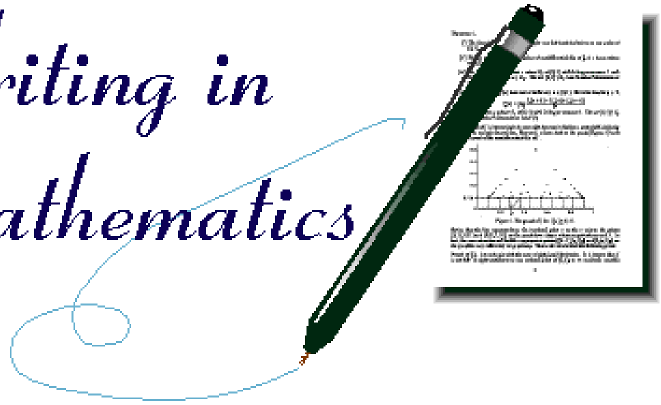
Writing in Mathematics



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1: Why Should You Have To Write Papers In A Math Class?

For most of your life so far, the only kind of writing you've done in math classes has been on homeworks and tests, and for most of your life you've explained your work to people that know more mathematics than you do (that is, to your teachers). But soon, this will change.

Now that you are taking Calculus, you know far more mathematics than the average American has ever learned - indeed, you know more mathematics than most college graduates remember. With each additional mathematics course you take, you further distance yourself from the average person on the street. You may feel like the mathematics you can do is simple and obvious (doesn't everybody know what a function is?), but you can be sure that other people find it bewilderingly complex. It becomes increasingly important, therefore, that you can explain what you're doing to others that might be interested: your parents, your boss, the media.

Nor are mathematics and writing far-removed from one another. Professional mathematicians spend most of their time writing: communicating with colleagues, applying for grants, publishing papers, writing memos and syllabi. Writing well is extremely important to mathematicians, since poor writers have a hard time getting published, getting attention from the Deans, and obtaining funding. It is ironic but true that most mathematicians spend more time writing than they spend doing math.

But most of all, one of the simplest reasons for writing in a math class is that writing helps you to learn mathematics better. By explaining a difficult concept to other people, you end up explaining it to yourself.

Every year, we buy ten cases of paper at $35 each; and every year we sell them for about $1 million each. Writing well is very important to us.

- Bill Browning, President of *Applied Mathematics, Inc.*

2: How is Mathematical Writing Different from What You've Done So Far?

A good mathematical essay has a fairly standard format. We tend to start solving a problem by first explaining what the problem is, often trying to convince others that it's an interesting or worthwhile problem to solve. On your homeworks, you've usually just said, "9(a)" and then plunged ahead; but in your formal writing, you'll have to take much greater pains.

After stating what the problem is, we usually then state the answer, even before we show how we got it. Sometimes we even state the answer right along with the problem. It's uncommon, although not so uncommon as to be exceptional, to read a math paper in which the answer is left for the very end. Explaining the solution and then the answer is usually reserved for cases where the solution technique is even more interesting than the answer, or when the writers want to leave the readers in suspense. But if the solution is messy or boring, then it's typically best to hook the readers with the answer before they get bogged down in details.

Another difference is that when you do your homework, it is important to show exactly how you got your answer. However, when you write to a non-mathematician, sometimes it's better to show why your answer works, with just a brief explanation as to how you got it. For example, compare:

Homework Mathematics:
To solve for x when $3x^2 - 21x + 30 = 0$, we use the quadratic formula:

$$\begin{aligned} x &= \frac{21 \pm \sqrt{21^2 - 4 \times 3 \times 30}}{2 \times 3} \\ &= \frac{21 \pm \sqrt{441 - 360}}{6} \\ &= \frac{21 \pm 9}{6} \\ &= \frac{30}{6} \text{ or } \frac{12}{6} \\ &= 5 \text{ or } 2, \end{aligned}$$

and so either $x = 5$ or $x = 2$.

More Formal Mathematics:
To solve for x when $3x^2 - 21x + 30 = 0$, we used the quadratic formula and found that either $x = 5$ or $x = 2$. It's easy to see that these are the right answers, because

$$(3)(5)^2 - (21)(5) + 30 = 75 - 105 + 30 = 0,$$

and also

$$(3)(2)^2 - (21)(2) + 30 = 12 - 42 + 30 = 0.$$

The difference is that, in the first example, you're trying to convince someone who knows a lot of math that you, too, know what you're doing (and if you don't, to get partial credit). In the second example, you're trying to show someone who may or may not be good at math that you got the right answer.

Math is difficult enough that the writing around it should be simple. 'Beautiful' math papers are the ones that are the easiest to read: clear explanations, uncluttered expositions on the page, well-organized presentation. For that reason, mathematical writing is not a creative endeavor the same way that, say, poetry is: you shouldn't be spending a lot of time looking for the perfect word, but rather should be developing the most clear exposition. Unlike humanities students, mathematicians don't have to worry about over-using 'trite' phrases in mathematics. In fact, at the end of this booklet are a list of trite but useful phrases that you may want to use in your papers, either in this class or in the future.

This guide, together with the checklist, should serve as a reference while you write. If you can master these basic areas, your writing may not be spectacular, but it should be clear and easy to read - which is the goal of mathematical writing, after all.

3: Following the Checklist

When you turn in your writing assignment, you should staple or use a paper clip to attach the checklist (available from the course web page) to the front. You should feel free to use both the checklist and this booklet as a guide while you write, because you will be graded directly on the criteria outlined on the checklist. What follows here is a more detailed explanation of the criteria used for grading your papers.

1. Clearly restate the problem to be solved.

Do not assume that the reader knows what you're talking about. (The person you're writing to might be out on vacation, for example, or have a weak memory). You don't have to restate every detail, but you should explain enough so that someone who's never seen the assignment can read your paper and understand what's going on, without any further explanation from you. Outline the problem carefully.

2. State the answer in a complete sentence which stands on its own.

If you can avoid variables in your answer, do so; otherwise, remind the reader what they stand for. If your answer is at the end of the paper and you've made any significant assumptions, restate them, too. Do not assume that the reader has actually read every word and remembers it all (do you?).

3. Clearly state the assumptions which underlie the formulas.

For example, what physical assumptions do you have to make? (No friction, no air resistance? That something is lying on its side, or far away from everything else?) Sometimes things are so straightforward that there are no assumptions, but not often.

4. Provide a paragraph which explains how the problem will be approached.

It's not polite to plunge into mathematics without first warning your reader. Carefully outline the steps you're going to take, giving some explanation of why you're taking that approach. It's nice to refer back to this paragraph once you're deep in the thick of your calculations.

5. Use diagrams, tables, or graphs, to help explain the math and clearly label them (if these are used).

In math, even more than in literature, a picture is worth a thousand words, especially if it's well labeled.

Label all axes, with words, if you use a graph. Give diagrams a title describing what they represent. It should be clear from the picture what any variables in the diagram should represent. The whole idea is to make everything as clear and self-explanatory as possible.

6. Define all variables used.

(a) Even if you label your diagram (and you should), you should still explain in words what your variables are.

(b) If there's a quantity you use only a few times, see if you can get away with *not* assigning it a variable. As examples:

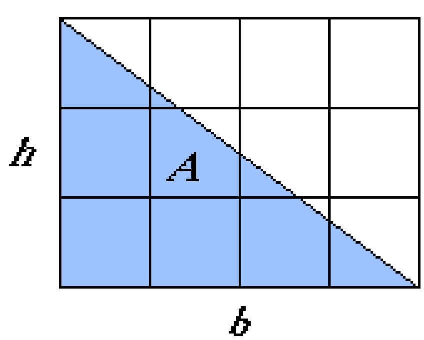
<table>
<tr>
<td rowspan="2">

Figure 1: ***Diagram of the triangle*** **(Each square is 1" by 1")**</td>
<td>We see that $A = (1/2)(h)(b)$, where A stands for the area of the triangle, b stands for the base of the triangle, and h stands for the height of the triangle, and so $A = (1/2)(3)(4) = 6$ square inches.</td>
<td>✗</td>
</tr>
<tr>
<td>We see that the area of the triangle will be one-half of the product of its height and base -- that is, the area of the triangle is $(1/2)(3)(4) = 6$ square inches.</td>
<td>✓</td>
</tr>
</table>

Elementary physics tells us that $v(t) = g(t - t_0)$, where $v(t)$ is the velocity of the falling object at time t, g is gravity, and t_0 is the time at which the object is released. Therefore as t increases, so does $v(t)$: i.e., as time increases, so does velocity.	✗
Elementary physics tells us that the velocity of a falling body is proportional to the amount of time it has already spent falling. Therefore, the longer it falls, the faster it goes.	✓

I hope that you'll agree that the second example of each pair is easier to read.

(c) The more specific you are, the better. State the units of measurement. When you can use words like "of", "from", "above", etc., do so. For example:

We get the equation $d = rt$, where d is the distance, r is the rate, and t is the time.	✗
We get the equation $d = rt$, where d is the distance from Sam's car to her home (in miles), r is the speed at which she's traveling (measured in miles per hour), and t is the number of hours she's been on the road.	✓

Avoid words like "position" (height above ground? sitting down? political situation?) and "time" (5 o'clock? January? 3 minutes since the experiment started?).

(d) Variables in text are *italicized* to tell them apart from regular letters.

7. Explain how each formula is derived, or where it can be found.

Don't pull formulas out of a hat, and don't use variables which you don't define. Either derive the formula yourself in the paper, or explain exactly where you found it, so other people can find it, too.

Put important or long formulas on a line of their own, and then center them; it makes them much easier to read:

The total number of infected cells in a honeycomb with n layers is 1+2+...+n = n(n+1)/2. Therefore, there are 100(101)/2=5,050 infected cells in a honeycomb with 100 layers.	✗
The total number of infected cells in a honeycomb with n layers is $$1 + 2 + ... + n = n(n + 1)/2.$$ Therefore, there are 100(101)/2 = 5,050 infected cells in a honeycomb with 100 layers.	✓

A Caution: Mathematical formulas are hard to do on a standard word processor. If you've got something that's really complicated, feel free to leave space and then write it out by hand later -- but then don't forget to do so!

8. Give acknowledgment where it is due.

Plagiarism is almost certainly the greatest sin in academia -- some fiction writers make plagiarism a motive for murder. It's extremely important to acknowledge where your inspiration, your proofreading, and your support came from. In particular, you should cite: any book you look at, any software which helped you understand or solve the problem, any student you talk to, and any professor you talk to (including me). The more specific you are, the better.

9. In this paper, are the spelling, grammar, and punctuation correct?

> (a) It may surprise you that it is on spelling and grammar that people tend to lose most of their points on their mathematics papers. Please spell-check and proofread your work for grammar mistakes. Better yet, ask a friend to read your paper. Mathematicians are generally not petty, but neither are we amused by sloppy or careless writing.

(b) Mathematical formulas are like clauses or sentences: they need proper punctuation, too. Put periods at the end of a computation if the computation ends the sentence; use commas if it doesn't. An example follows.

If Dr. Crannell's caffeine level varies proportionally with time, we see that $$C(t) = kt,$$ where $C(t)$ is her caffeine level t minutes after 7:35 a.m., and k is a constant of proportionality. We can solve to show that k = 202, and therefore her caffeine level by 11:02 (t =207) is $$\begin{aligned} \mathrm{C}(202) &= (202)(207) \\ &= 41{,}814. \end{aligned}$$	✓

(c) Do not confuse mathematical symbols for English words (= and # are especially common examples of this). The symbol "=" is used only in mathematical formulas -- not in sentences:

We let V = volume of a single mug and n = the # of mugs. Then the formula for the total amount of root beer $R = nV$.	✗
We let V stand for the volume of the mug and n represent the number of mugs. Then the formula for the total amount of root beer we can pour, R, is R is nV.	✗
We let V stand for the volume of a single mug and n represent the number of mugs. Then the formula for the total amount of root beer we can pour, R, is $R = nV$.	✓

(d) Do, however, use equal signs when you state formulas or equations, because mathematical sentences need subjects and verbs, too.

Then the formula for the total amount of root beer we can pour is nV.	✗
Then the formula for the total amount of root beer we can pour is $R = nV$.	✓

(e) Whether or not you use a computer, *please* proofread the final draft before you give it to me.

10. In this paper, is the mathematics correct?

This is self-explanatory.

11. In this paper, did the writer solve the question that was originally asked?

So is this.

4: Good Phrases to Use in Math Papers:

- *Therefore* (also: *so, hence, accordingly, thus, it follows that, we see that, then*)
- *I am assuming that* (also: *assuming, where, M stands for)*
- *show* (also: *demonstrate, prove, explain why, find*)
- *This formula can be found on page 9-743 of Discovering Calculus ©1999, Levine.*
- (*using the formula above*). (also: *see the forumla above, this tells us that ...*)
- *if* (also: *whenever, provided that, when*)
- *notice that* (also: *note that, notice, recall*)
- *since* (also: *because*)

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