February 15, 2008

Written Problem #6

To find the volume of the first cake I used part of the equation for the base. The baker stated that the base is formed by $y = \sin x$, $y = -\sin x$ and $0 \le x \le \pi$ so therefore the base of the graph is symmetrical. That said, to find the volume we can use $y = \sin x$ alone as opposed to the full equation of the base.

Also, it must be noted that the slices of the cake form perfect semicircles so the volume equation must also be divided by 2.

$$V = \int_a^b \frac{\pi}{2} r^2 \, dx$$

$$V = \int_0^\pi \frac{\pi}{2} (\sin x)^2 \, dx$$

$$V = \frac{\pi}{2}\left[\frac{1}{2}x - \frac{1}{4}\sin(2x)\right]_0^\pi$$

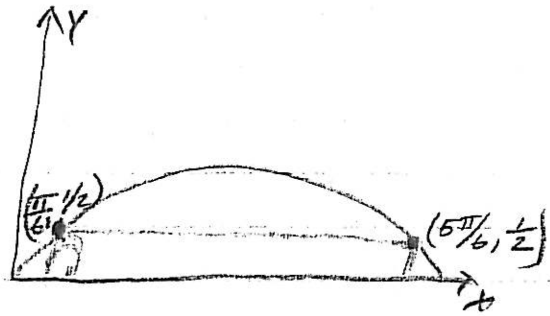
$$V = \frac{\pi}{2}\left[\frac{\pi}{2} - \frac{1}{4}\sin(2\pi)\right]$$

$$V = \frac{\pi}{2}\left(\frac{\pi}{2}\right)$$

$V = \frac{\pi^2}{4}$ = volume of plain chocolate cake

The volume of the second cake with the ice cream filling has to be found using the washer method. The ice cream center has a radius of 1/2 because it extends half way towards the edge of the cake.

Also since we've been told that the cylinder is cut off towards the ends of the cake, we have to find where exactly it is cut off in order to find its volume. To find a and b of its integral we solve $\sin(x)=\frac{1}{2}$ for x.



$$\sin(x) = \frac{1}{2} \qquad 0 \leq x \leq \pi$$
$$x = \pi/6 \,,\ \frac{5\pi}{6}$$

Now we can find the volume of the ice cream cylinder and subtract it from the volume of the cake.

Volume of the cylinder

$$V = \int_{\pi/6}^{5\pi/6} \frac{\pi}{2}\left(\frac{1}{2}\right)^2 dx$$

$$V = \frac{\pi}{2}\int_{\pi/6}^{5\pi/6} \frac{1}{4}\, dx$$

$$V = \frac{\pi}{2}\left[\frac{1}{4}x\right]_{\pi/6}^{5\pi/6}$$

$$V = \frac{\pi}{2}\left[\frac{5\pi}{24} - \frac{\pi}{24}\right]$$

$$V = \frac{\pi}{2}\left(\frac{\pi}{6}\right)$$

$$V = \frac{\pi^2}{12}$$

With the volume of the cylinder known we can subtract it from the volume of the cake which is $\pi^2/4$

Volume of second cake eaten =

$$\frac{\pi^2}{4} - \frac{\pi^2}{12}$$

$$= \frac{\pi^2}{6}$$