Written Problem 3

Since you are doing so well in Math 2 this term, you are being interviewed for a position as an undergraduate study group leader for next year's Math 1/2 classes. As part of your interview process the hiring committee has presented you with the problem below. Your instructions are to write up a solution to the problem.

Suppose Molly is filling up her tub for a nice, warm, relaxing bath. It takes a while for the water to be hot enough, so in the beginning the flow of hot water is slow, but it increases until it reaches 4 gallons of hot water per minute. She fills it at that rate for three more minutes, and then someone in her building must have started their laundry, because the flow of hot water decreased pretty rapidly and then settled at a slow two gallons per minute. She continued at this rate until her 25 gallon bath tub was full. The function $r(t)$ below gives the the rate of flow of hot water into Molly's tub in gallons per minute.

$$r(t) = \begin{cases} 2t & \text{if } 0 \leqslant t < 2 \\ 4 & \text{if } 2 \leqslant t < 5 \\ 14 - 2t & \text{if } 5 \leqslant t < 6 \\ 2 & \text{if } 6 \leqslant t < t_{\text{full}} \end{cases}$$

1. *Draw a carefully labeled graph of this function.*

2. *What does the function $w(T) = \int_0^T r(t)\, dt$ for $T > 0$ represent? What are its units?*

3. *Use geometry to find an expression of $w(T)$. (This will be a piecewise representation, similar to that of $r(t)$. You may find the formula for the area of a trapezoid useful: $A = \frac{h}{2}(b_1 + b_2)$, where b_1 and b_2 are the lengths of the parallel sides, and h represents the height.)*

4. *Use your expression for $w(T)$ to find t_{full}, the time Molly shuts off the water because her tub is full.*

Remember, your solution should be geared towards a struggling Math 2 student. It should, therefore, include **all** your reasoning and **clear** explanations of the steps you took to solve the problem. Pretend like you are explaining the solution to a student. Write down everything you would say, and try to think where they might have trouble or questions, and include answers or tips in your written solution.