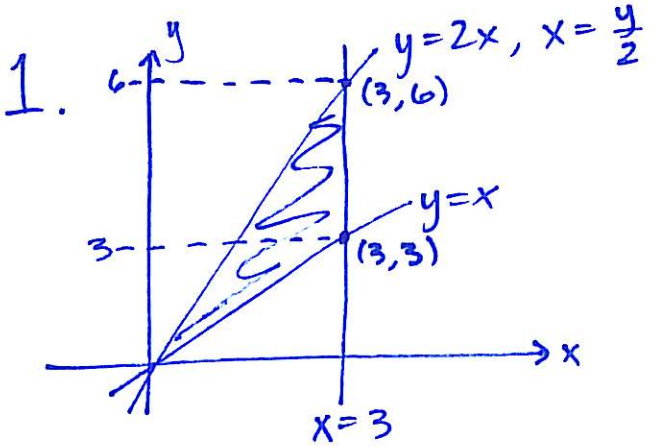
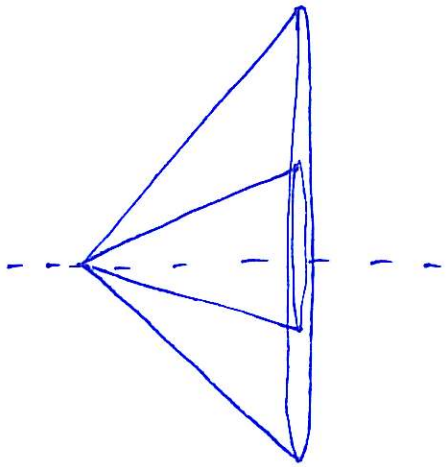
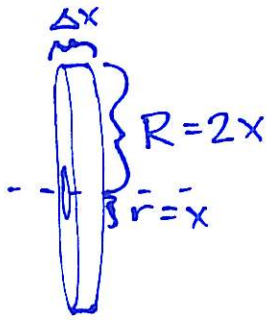
Quiz 5 Solutions

1.





a) WASHERS



$V_{washer} = \pi(R^2 - r^2)\Delta x$

$= \pi((2x)^2 - (x)^2)\Delta x$

$= \pi(4x^2 - x^2)\Delta x$

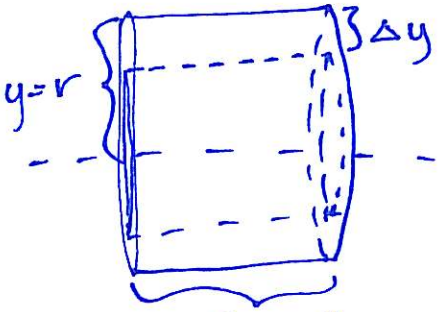
$= \pi(3x^2)\Delta x$

$V = \int_0^3 3\pi x^2 dx$

$= \pi\left[x^3\right]_0^3$

$= \boxed{27\pi}$

b) SHELLS



$h = \begin{cases} y - \frac{y}{2} & 0 \le y \le 3 \\ 3 - \frac{y}{2} & 3 \le y \le 6 \end{cases}$

$V_{shell} = 2\pi r h \Delta y$

$= \begin{cases} 2\pi y\left(y - \frac{y}{2}\right)\Delta y \\ 2\pi y\left(3 - \frac{y}{2}\right)\Delta y \end{cases} = \begin{cases} 2\pi\left(\frac{y^2}{2}\right)\Delta y \\ 2\pi\left(3y - \frac{y^2}{2}\right)\Delta y \end{cases}$

$V = \int_0^3 2\pi \frac{y^2}{2} dy + \int_3^6 2\pi\left(3y - \frac{y^2}{2}\right) dy$

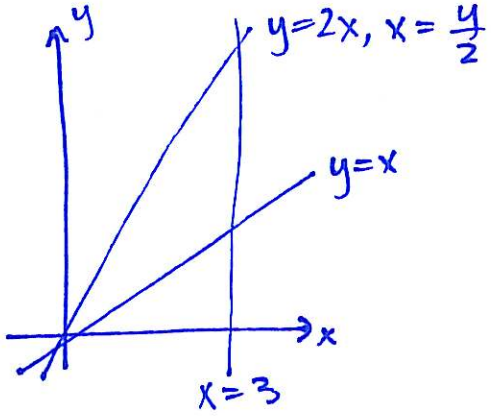
$= \pi\left[\frac{y^3}{3}\right]_0^3 + 2\pi\left[\frac{3y^2}{2} - \frac{y^3}{6}\right]_3^6$

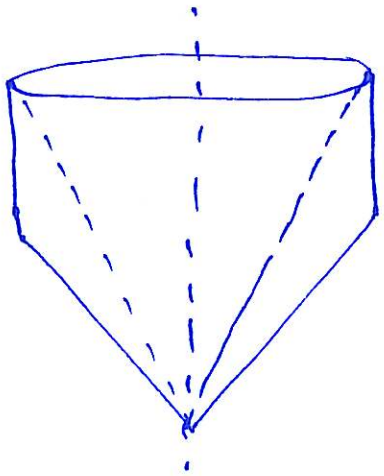
$= \pi\left(\frac{27}{3} - 0\right) + 2\pi\left[\left(\frac{3 \cdot 36}{2} - \frac{63}{6}\right) - \left(\frac{27}{2} - \frac{27}{6}\right)\right]$

$= \pi\left[9 + 108 - 72 - 27 + 9\right]$

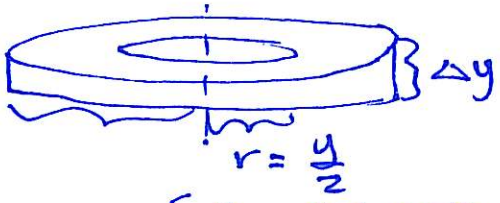
$= \boxed{27\pi}$

2.





a) WASHERS



$$R = \begin{cases} y & 0 \le y \le 3 \\ 3 & 3 \le y \le 6 \end{cases}$$

$$V_{washer} = \pi(R^2 - r^2)\Delta y$$

$$= \begin{cases} \pi\left(y^2 - \left(\frac{y}{2}\right)^2\right)\Delta y \\ \pi\left(3^2 - \left(\frac{y}{2}\right)^2\right)\Delta y \end{cases}$$

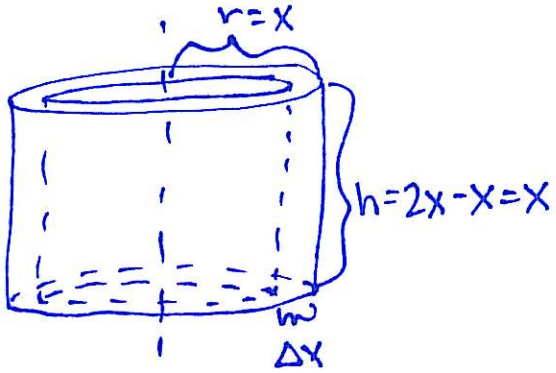
$$= \begin{cases} \pi\left(\frac{3y^2}{4}\right)\Delta y \\ \pi\left(9 - \frac{y^2}{4}\right)\Delta y \end{cases}$$

$$V = \int_0^3 \pi \frac{3y^2}{4}\,dy + \int_3^6 \pi\left(9 - \frac{y^2}{4}\right)dy = \pi\left[\frac{y^3}{4}\right]_0^3 + \pi\left[9y - \frac{y^3}{12}\right]_3^6$$

$$= \pi\left(\frac{27}{4} - 0\right) + \pi\left[\left(54 - \frac{6^3}{12}\right) - \left(27 - \frac{27}{12}\right)\right]$$

$$= \pi\left(\frac{27}{4} + 27 + \frac{27 - 216}{12}\right) = \pi\left(27 + \frac{27}{4} - \frac{63}{4}\right) = \pi\left(27 - \frac{36}{4}\right) = \boxed{18\pi}$$

b) SHELLS



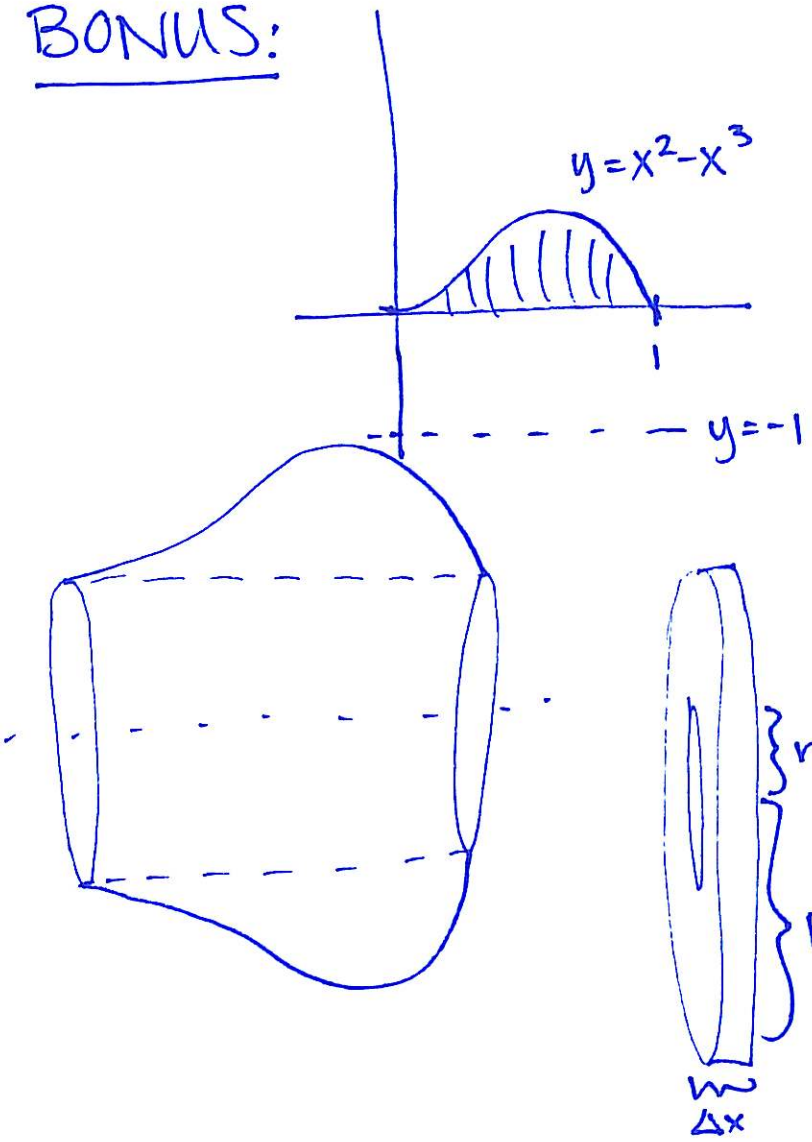
$$V_{shell} = 2\pi r h \Delta x$$

$$= 2\pi(x)(x)\Delta x = 2\pi x^2 \Delta x$$

$$V = \int_0^3 2\pi x^2\,dx = 2\pi \frac{x^3}{3}\Big|_0^3$$

$$= 2\pi\left(\frac{27}{3}\right) = \boxed{18\pi}$$

BONUS:



$x^2 - x^3 = 0$

$x^2(1 - x) = 0$

$x = 0, 1$

$V_{washer} = \pi(R^2 - r^2)\Delta x$

$= \pi\left((1 + x^2 - x^3)^2 - 1^2\right)\Delta x$

$$V = \int_0^1 \pi\left((1 + x^2 - x^3)^2 - 1\right) dx$$

$$= \pi \int_0^1 1 + x^2 - x^3 + x^2 + x^4 - x^5 - x^3 - x^5 + x^6 - 1 \, dx$$

$$= \pi \int_0^1 x^6 - 2x^5 + x^4 - 2x^3 + 2x^2 \, dx$$

$$= \pi \left[\frac{x^7}{7} - \frac{2x^6}{6} + \frac{x^5}{5} - \frac{2x^4}{4} + \frac{2x^3}{3}\right]_0^1$$

$$= \pi\left(\frac{1}{7} - \frac{1}{3} + \frac{1}{5} - \frac{1}{2} + \frac{2}{3}\right) = \pi\left(\frac{30 - 70 + 42 - 105 + 140}{210}\right)$$

$$= \frac{37}{210}\pi$$