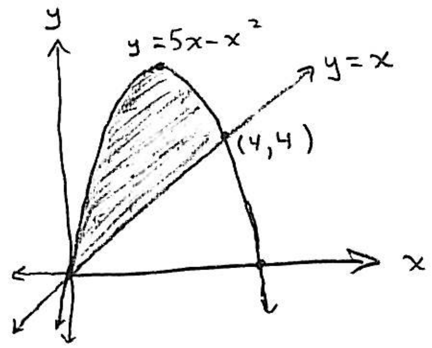
MATH 2 SOLUTIONS TO PROBLEM SET # 9

SECTION 6.1 STEWART

(1.)



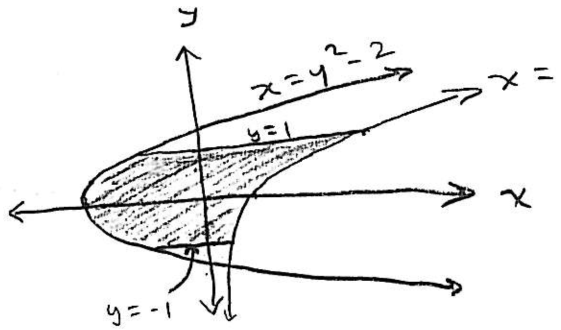
$$A = \int_0^4 |5x - x^2 - x|\, dx$$

$$= \int_0^4 (5x - x^2 - x)\, dx$$

$$= \int_0^4 (4x - x^2)\, dx$$

$$= \left[2x^2 - \frac{1}{3}x^3 \right]_0^4 = \boxed{\frac{32}{3}}.$$

(3.)



$$A = \int_{-1}^1 |y^2 - 2 - e^y|\, dy$$

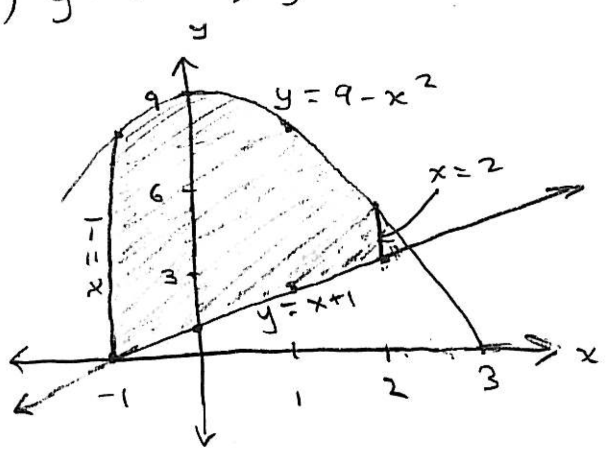
$$= \int_{-1}^1 (e^y - y^2 + 2)\, dy$$

$$= \left[e^y - \frac{1}{3}y^3 + 2y \right]_{-1}^1$$

$$= \left(e - \frac{1}{3} + 2\right) - \left(\frac{1}{e} + \frac{1}{3} - 2\right)$$

$$= e - \frac{1}{e} - \frac{2}{3} + 4 = \boxed{e - \frac{1}{e} + \frac{10}{3}}$$

(5.) $y = x + 1$, $y = 9 - x^2$, $x = -1$, $x = 2$



$$A = \int_{-1}^2 |(9 - x^2) - (x+1)|\, dx$$

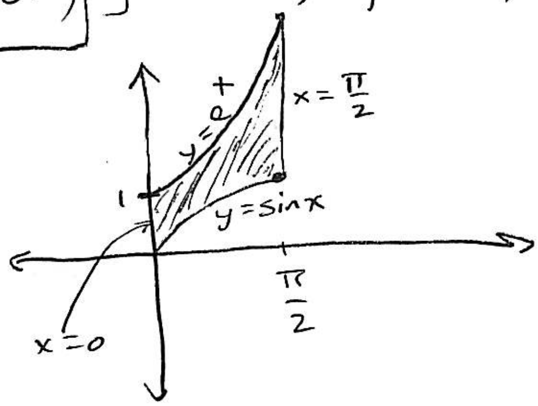
$$= \int_{-1}^2 (9 - x^2) - (x + 1)\, dx$$

$$= \int_{-1}^2 (-x^2 - x + 8)\, dx$$

$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 8x \right]_{-1}^2$$

$$= \left(-\frac{8}{3} - 2 + 16\right) - \left(\frac{1}{3} - \frac{1}{2} - 8\right) = \boxed{19.5}.$$

(6.) $y = \sin x, \quad y = e^x, \quad x = 0, \quad x = \frac{\pi}{2}$

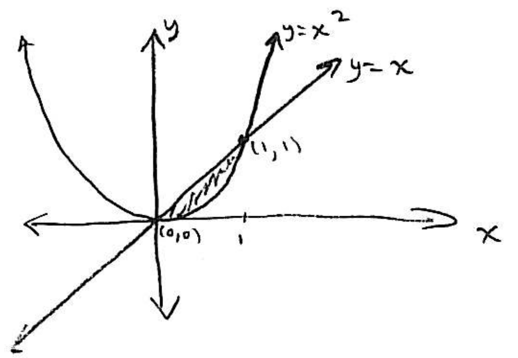


$$A = \int_0^{\frac{\pi}{2}} |\sin x - e^x| \, dx =$$

$$\int_0^{\frac{\pi}{2}} (e^x - \sin x) \, dx$$

$$= \left[e^x + \cos x \right]_0^{\frac{\pi}{2}} = \boxed{e^{\pi/2} - 2}.$$

(7.) $y = x, \quad y = x^2$



$x^2 = x \iff x^2 - x = 0$

$\iff x(x-1) = 0 \iff x = 0$ OR 1.

THUS THE INTERSECTION POINTS ARE $(0,0)$, $(1,1)$.

$$A = \int_0^1 |x^2 - x| \, dx = \int_0^1 (x - x^2) \, dx$$

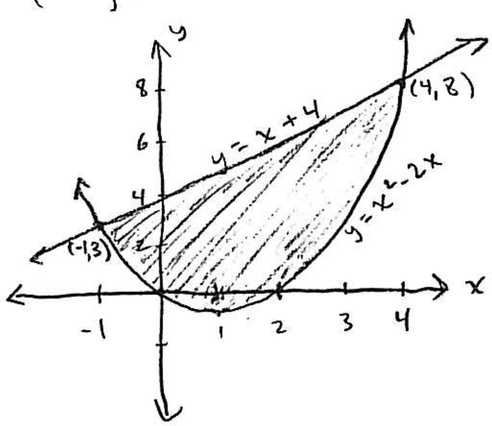
$$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}.$$

(8.) $y = x^2 - 2x, \quad y = x + 4$

$x^2 - 2x = x + 4 \iff x^2 - 3x - 4 = 0 \iff (x-4)(x+1) = 0$

$\iff x = -1$ OR 4, SO THE INTERSECTION POINTS ARE

$(-1, 3)$ AND $(4, 8)$.



$$A = \int_{-1}^4 |(x^2 - 2x) - (x + 4)| \, dx$$

$$= \int_{-1}^4 (x+4) - (x^2 - 2x) \, dx$$

$$= \int_{-1}^4 (-x^2 + 3x + 4) \, dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{3}{2}x^2 + 4x \right]_{-1}^4$$

$$= \left(-\frac{64}{3} + 24 + 16 \right) - \left(\frac{1}{3} + \frac{3}{2} - 4 \right)$$

$$= 42.5 - \frac{65}{3} = \boxed{\frac{125}{6}}.$$