# MATH 2 : SOLUTIONS TO PROBLEM SET # 5

NOTE: I TOLD THE STUDENTS THEY COULD SAVE A LOT OF TIME ON # 23, # 26c BY INCLUDING A PROOF (FROM THE DEFINITION OF DEFINITE INTEGRAL AS THE LIMIT OF A RIEMANN SUM) THAT $\int_a^b x^2\,dx = \frac{b^3-a^3}{3}$, BASED ON A WORKSHEET THEY HAVE, AND THEN USING THE INTEGRAL LAWS, SO AS TO AVOID DOING THIS NASTY COMPUTATION TWICE OVER. (SINCE 26c IS ONLY WORTH ABOUT A QUARTER OF A POINT,) IF THEY MESS UP ON PROVING $\int_a^b x^2\,dx = \frac{b^3-a^3}{3}$ BUT APPLY IT CORRECTLY, YOU COULD GO AHEAD AND GIVE THEM CREDIT FOR 26c.

<u>CLAIM</u>: $\int_a^b x^2\,dx = \frac{b^3-a^3}{3}$.

<u>PROOF</u>:

$$\int_a^b x^2\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left(a+k\left(\frac{b-a}{n}\right)\right)^2\left(\frac{b-a}{n}\right)$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\sum_{k=1}^{n}\left(a^2+\frac{2a(b-a)}{n}k+\left(\frac{b-a}{n}\right)^2k^2\right)$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\left(na^2+\frac{2a(b-a)}{n}\cdot\frac{n(n+1)}{2}+\left(\frac{b-a}{n}\right)^2\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty}\left(a^2(b-a)+a(b-a)^2\left(1+\frac{1}{n}\right)+\frac{(b-a)^3}{6}\cdot\frac{n(n+1)(2n+1)}{n^3}\right)$$

$$= a^2(b-a)+a(b-a)^2+\frac{(b-a)^3}{6}\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)$$

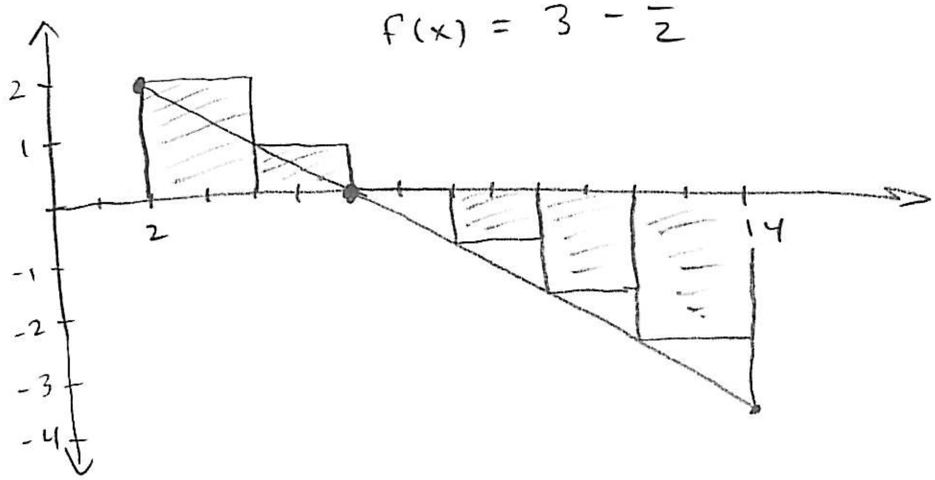
$$= a^2(b-a)+a(b-a)^2+\frac{(b-a)^3}{3}$$

$$= \cancel{a^2b}-\cancel{a^3}+\cancel{ab^2}-\cancel{2a^2b}+\cancel{a^3}+\frac{b^3}{3}-\cancel{ab^2}+\cancel{a^2b}-\frac{a^3}{3}$$

$$= \frac{b^3-a^3}{3}. \qquad \blacksquare$$

# SECTION 5.2

(1.) $f(x) = 3 - \frac{x}{2}$




$$L_6 = 2\left(2 + 1 + 0 + (-1) + (-2) + (-3)\right) = \boxed{-6}$$

(SUM OF AREAS OF FIRST TWO RECTANGLES, MINUS SUM OF AREAS OF LAST FOUR RECTANGLES. OVERESTIMATE OF THE ACTUAL NET AREA.)

(21.) $$\int_{-1}^{5} (1+3x)\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + 3\left(-1 + k\left(\frac{6}{n}\right)\right)\right)\frac{6}{n}$$

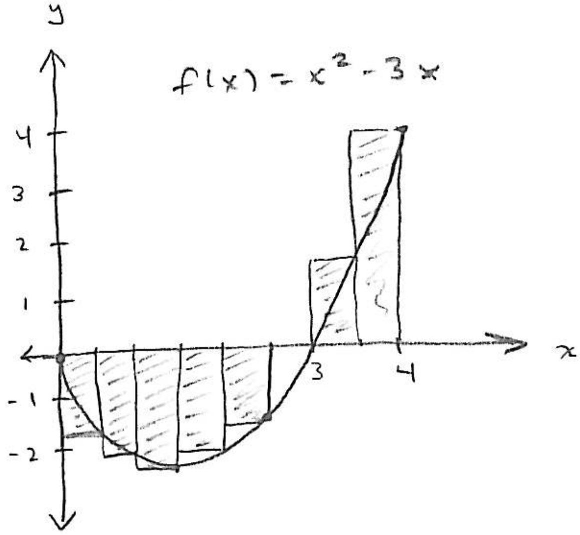
$$= \lim_{n\to\infty} \frac{6}{n}\sum_{k=1}^{n}\left(-2 + \frac{18}{n}k\right) = \lim_{n\to\infty} \frac{6}{n}\left(-2n + \frac{18}{n}\cdot\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(-12 + 54\left(1 + \frac{1}{n}\right)\right) = -12 + 54 = \boxed{42}.$$

(23.) BY CLAIM AND INTEGRAL LAWS,

$$\int_0^2 (2 - x^2)\, dx = \int_0^2 2\, dx - \int_0^2 x^2\, dx = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}.$$

(26.) (a-b.)




$$R_8 = \frac{1}{2}\left(-\frac{5}{4} - 2 - \frac{9}{4} - 2 - \frac{5}{4} + 0 + \frac{7}{4} + 4\right) = \boxed{-1.5}$$

(c.) $\int_0^4 (x^2 - 3x)\,dx = \int_0^4 x^2\,dx - 3\int_0^4 x\,dx$ (INTEGRAL LAWS)

$= \dfrac{4^3 - 0^3}{3} - 3\int_0^4 x\,dx$ (BY CLAIM)

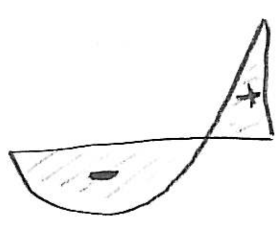
$= \dfrac{64}{3} - 3\int_0^4 x\,dx.$

NOW, $\int_0^4 x\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{4k}{n}\cdot\frac{4}{n} = \lim_{n\to\infty} \frac{16}{n^2}\sum_{k=1}^{n} k$

$= \lim_{n\to\infty} \frac{16}{n^2}\cdot\frac{n(n+1)}{2} = 8\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 8,$

THUS $\int_0^4 (x^2 - 3x)\,dx = \frac{64}{3} - 3(8) = \boxed{-\frac{8}{3}}.$

(d.) $\int_0^4 (x^2 - 3x)\,dx$ = NET AREA UNDER $f(x) = x^2 - 3x$ FROM 0 TO 4 = AREA OF PART ABOVE X-AXIS MINUS AREA OF PART BELOW X-AXIS.



(43.) $\int_0^1 (5-6x^2)\,dx = \int_0^1 5\,dx - 6\int_0^1 x^2\,dx$

$= 5(1-0) - 6\left(\frac{1}{3}\right) = \boxed{3}.$

$\boxed{(44.)}$ $\int_1^3 (2e^x - 1)\,dx = 2\int_1^3 e^x\,dx - \int_1^3 1\,dx$

$= 2(e^3 - e) - 1(3-1) = \boxed{2(e^3 - e - 1)}.$