MATH 2: SOLUTIONS TO PROBLEM SET #1

REVIEW EXERCISES FOR CHAPTER #1:

(2.) (a.) $g(2) = 3$

(b.) g IS 1-1 (i.e. $x \neq y \Rightarrow g(x) \neq g(y)$)

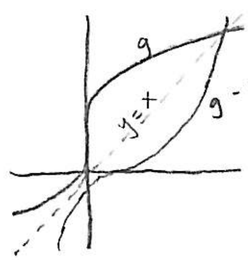
BECAUSE IT PASSES THE HORIZONTAL LINE TEST. (SEE p. 60.)

(c.) $g^{-1}(2)$ IS ABOUT $\frac{1}{4}$

SINCE $g(x) = 2$ WHEN x IS ABOUT $\frac{1}{4}$.

(d.) THE DOMAIN OF g^{-1} IS (APPROXIMATELY) $[-1, 3.5]$.

(e.) THE GRAPH OF g^{-1} IS THE GRAPH OF g REFLECTED ABOUT THE LINE $y = x$.



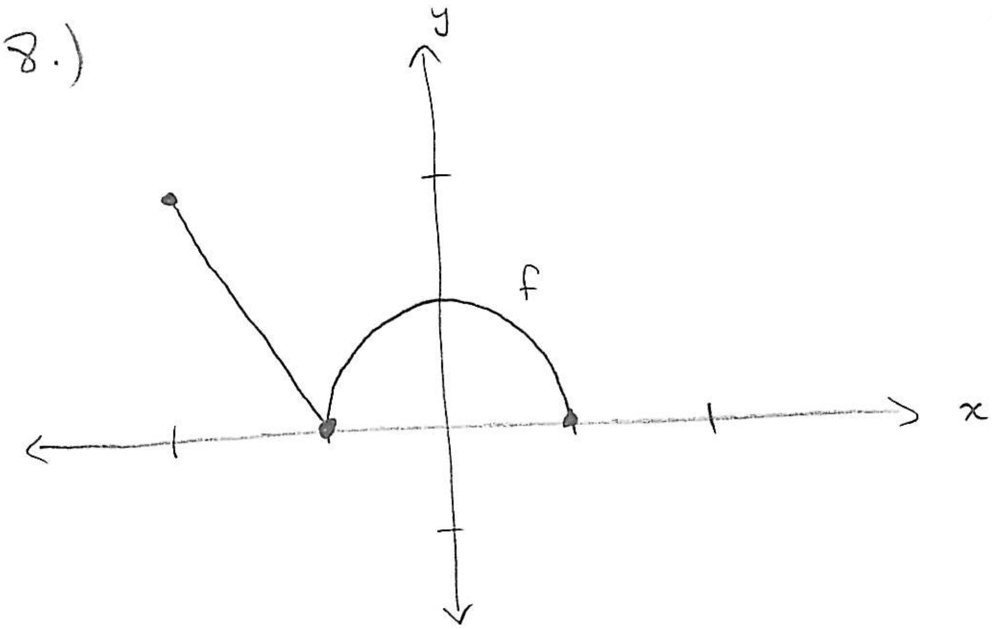
(8.) $F(t) = 3 + \cos(2t)$ IS DEFINED EVERYWHERE, SO THE DOMAIN IS ALL REAL NUMBERS, $\mathbb{R}$.

$\cos(2t)$ TAKES VALUES ON $[-1, 1]$

SO $3 + \cos(2t)$ TAKES VALUES ON $[2, 4]$

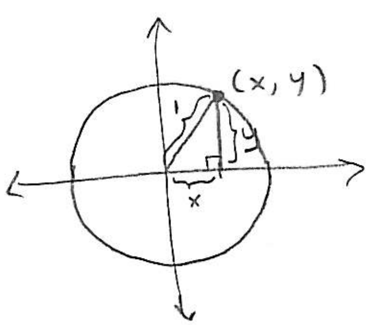
THUS THE RANGE IS $[2, 4]$.

(18.)



$$f(x) = \begin{cases} -2x - 2 & \text{IF } -2 \le x \le -1 \\ \sqrt{1 - x^2} & \text{IF } -1 \le x \le 1 \end{cases}$$

(SINCE $-2x-2$ IS THE EQUATION OF THE LINE WITH SLOPE -2, y-INTERCEPT -2, AND THE EQUATION OF THE CIRCLE OF RADIUS 1 CENTERED AT THE ORIGIN IS $x^2 + y^2 = 1$, BY THE PYTHAGOREAN THEOREM.)



(20.) $F(x) = \dfrac{1}{\sqrt{x+\sqrt{x}}}$

$F = f \circ g \circ h$

WHERE $h(x) = \sqrt{x}$,

$g(x) = \sqrt{x^2 + x}$,

$f(x) = 1/x$.

(22.) (a.) LET y = COST (DOLLARS)

x = TOASTER OVENS PER WEEK

WE WANT AN EQUATION FOR THE LINE PASSING THROUGH (1000, 9000) AND (1500, 12000), USE THE POINT-SLOPE FORMULA.

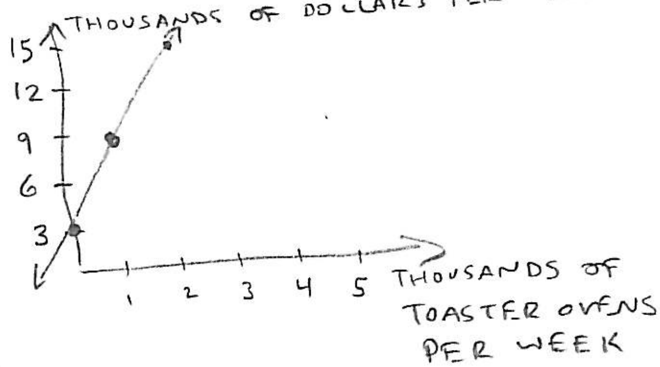
THE LINE HAS SLOPE $\dfrac{12000-9000}{1500-1000} = 6$

AND PASSES THROUGH (1000, 9000)

SO THE EQUATION IS

$y - 9000 = 6(x - 1000)$, OR

$y = 6x + 3000$.



(b.) THE SLOPE IS 6 (SEE ABOVE) AND IT REPRESENTS THE FACT THAT EVERY NEW PER WEEK TOASTER OVEN PRODUCED COSTS AN EXTRA \$6 PER WEEK.

(c.) \$3000, STARTING COSTS.

(26.) (a.) $e^x = 5$

$x = \ln(5)$ (TAKE NATURAL LOG OF BOTH SIDES.)

(b.) $\ln x = 2$

$x = e^2$ (EXPONENTIATE BOTH SIDES.)

(c.) $e^{e^x} = 2$

$e^x = \ln 2$ (TAKE NATURAL LOG OF BOTH SIDES.)

$x = \ln(\ln 2)$ (REPEAT.)

(d.) $\tan^{-1} x = 1$

$x = \tan(1)$ (TAKE TANGENT OF BOTH SIDES.)

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REVIEW EXERCISES FOR CHAPTER # 2 :

(8.) $$\frac{t^2-4}{t^3-8} = \frac{(t+2)(t-2)}{(t^2+2t+4)(t-2)}$$

$$= \frac{t+2}{t^2+2t+4} \quad \text{(PROVIDED } t \neq 2\text{)}$$

SO $$\lim_{t\to 2} \frac{t^2-4}{t^3-8} = \lim_{t\to 2} \frac{t+2}{t^2+2t+4} = \frac{4}{12} = \frac{1}{3}.$$

SINCE THE LIMIT DEPENDS ON THE BEHAVIOUR FOR t NEAR 2 (NOT AT 2).

LIMIT LAW FOR RATIONAL FUNCTIONS.

(10.) $$\lim_{v\to 4^+} \frac{4-v}{|4-v|} = \lim_{v\to 4^+} \frac{4-v}{-(4-v)} = -1$$

SINCE THE LIMIT DEPENDS ON THE BEHAVIOUR FOR V NEAR AND ABOVE 4 (NOT AT OR LESS THAN 4.)

SINCE $\frac{4-v}{-(4-v)} = -1$ FOR ALL $v \neq 4$

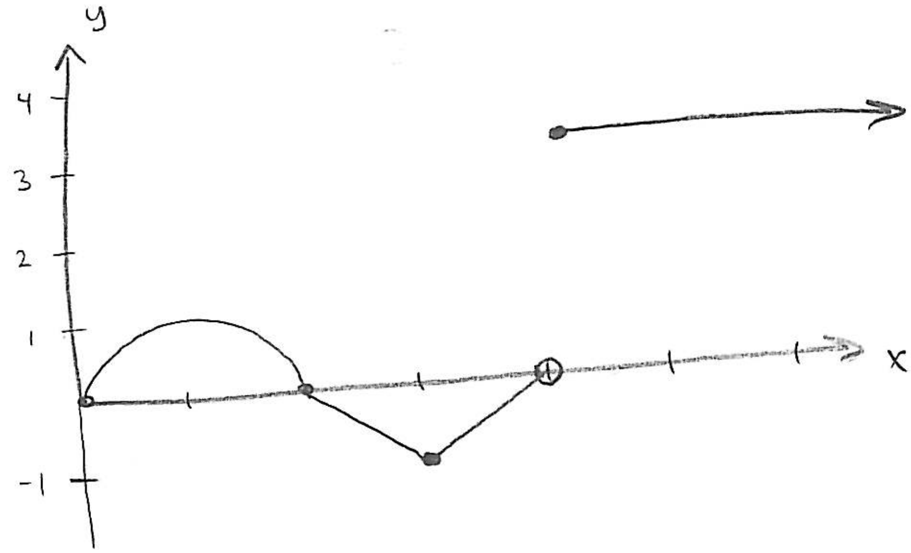
(16.) $$\lim_{x \to -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^4}$$

$$= \lim_{x \to -\infty} \frac{x^4 + 2x^2 - 1}{3x^4 - x - 5}$$ (MULTIPLY TOP & BOTTOM BY -1)

$$= \lim_{x \to -\infty} \frac{1 + \frac{2}{x^2} - \frac{1}{x^4}}{3 - \frac{1}{x^3} - \frac{5}{x^4}}$$ (DIVIDE TOP & BOTTOM BY x^4)

$= \frac{1}{3}$ (BY LIMIT LAWS, SINCE $\lim_{x \to -\infty} \frac{1}{x^2} = \lim_{x \to -\infty} \frac{1}{x^3} = \lim_{x \to -\infty} \frac{1}{x^4} = 0.$)

(30.)



- g CONTINUOUS AT 2. $\left(\lim_{x \to 2} g(x) = 0 = g(2)\right)$
- g CONTINUOUS AT 3. $\left(\lim_{x \to 3} g(x) = -1 = g(3)\right)$
- g IS ONLY CONTINUOUS FROM THE RIGHT AT 4. $\left(\lim_{x \to 4^+} g(x) = \pi = g(4), \lim_{x \to 4^-} g(x) = 0 \neq g(4)\right)$

(32.) $g(x) = \dfrac{\sqrt{x^2 - 9}}{x^2 - 2}$

DOMAIN IS $(-\infty, -3] \cup [3, \infty)$.

HERE, g CONTINUOUS SINCE IT CAN BE OBTAINED FROM CONSTANT FUNCTIONS AND THE FUNCTION x (WHICH ARE CONTINUOUS) BY ARITHMETIC AND TAKING SQUARE ROOTS, ALL OPERATIONS WHICH PRESERVE CONTINUITY WHERE DEFINED.