MATH 2 SOLUTIONS TO PROBLEM SET #14

SECTION 7.1 : INTEGRATION BY PARTS

$$\boxed{\int u\, dv = uv - \int v\, du}$$

(1.) $\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\, dx$

$u = \ln x \qquad v = \frac{1}{3}x^3$

$dv = x^2\, dx \qquad du = \frac{1}{x}\, dx$

$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$. (CHECK BY DIFFERENTIATION).

(5.) $\int r e^{r/2}\, dr$

$u = r \qquad v = 2e^{r/2}$

$dv = e^{r/2}\, dr \qquad du = dr$

$= 2re^{r/2} - \int 2e^{r/2}\, dr = \boxed{2re^{r/2} - 4e^{r/2} + C}$.

(6.) $\int t \sin 2t \, dt = -\frac{1}{2}t\cos(2t) - \int -\frac{1}{2}\cos(2t)\, dt$

$u = t \qquad v = -\frac{1}{2}\cos(2t)$

$dv = \sin(2t)\, dt \qquad du = dt$

$= -\frac{1}{2}t\cos(2t) + \frac{1}{2}\int \cos(2t)\, dt$

$= \boxed{-\frac{1}{2}t\cos(2t) + \frac{1}{4}\sin(2t) + C}$.

(19.) $\int_0^\pi t\sin(3t)\,dt = \left[-\frac{1}{3}t\cos(3t)\right]_0^\pi - \int_0^\pi -\frac{1}{3}\cos(3t)\,dt$

$$\boxed{\begin{array}{ll} u = t & v = -\frac{1}{3}\cos(3t) \\ dv = \sin(3t)\,dt & du = dt \end{array}}$$

$= -\frac{\pi}{3}\cos(3\pi) + \frac{1}{3}\int_0^\pi \cos(3t)\,dt$

$= \frac{\pi}{3} + \underbrace{\frac{1}{3}\left[\frac{1}{3}\sin(3t)\right]_0^\pi}_{0} = \boxed{\frac{\pi}{3}}$.

$\boxed{(20.)}$ $\int_0^1 (x^2+1)e^{-x}\,dx = \int_0^1 (x^2e^{-x} + e^{-x})\,dx$

$= \int_0^1 x^2e^{-x}\,dx + \int_0^1 e^{-x}\,dx$

$= \int_0^1 x^2e^{-x}\,dx + \left[-e^{-x}\right]_0^1 = \int_0^1 x^2e^{-x}\,dx + \left(1-\frac{1}{e}\right)$

$$\boxed{\begin{array}{ll} u = x^2 & v = -e^{-x} \\ dv = e^{-x}\,dx & du = 2x\,dx \end{array}}$$

$= \left[-x^2e^{-x}\right]_0^1 - \int_0^1 -e^{-x}\cdot 2x\,dx + \left(1-\frac{1}{e}\right)$

$= 2\int_0^1 xe^{-x}\,dx + \left(1-\frac{2}{e}\right)$

$$\boxed{\begin{array}{ll} u = x & v = -e^{-x} \\ dv = e^{-x}\,dx & du = dx \end{array}}$$

$= 2\left(\left[-xe^{-x}\right]_0^1 - \int_0^1 -e^{-x}\,dx\right) + \left(1-\frac{2}{e}\right)$

$= 2\int_0^1 e^{-x}\,dx + \left(1-\frac{4}{e}\right) = 2\left[-e^{-x}\right]_0^1 + \left(1-\frac{4}{e}\right)$

$2\left(1-\frac{1}{e}\right) + \left(1-\frac{4}{e}\right) = \boxed{3-\frac{6}{e}}$.

(25.) $\int_0^1 \frac{y}{e^{2y}}\,dy = \int_0^1 y e^{-2y}\,dy$

$$\boxed{\begin{array}{ll} u = y & v = -\frac{1}{2}e^{-2y} \\ dv = e^{-2y}\,dy & du = dy \end{array}}$$

$$= \left[-\frac{1}{2} y e^{-2y}\right]_0^1 - \int_0^1 -\frac{1}{2} e^{-2y}\,dy$$

$$= -\frac{1}{2e^2} + \frac{1}{2}\int_0^1 e^{-2y}\,dy = -\frac{1}{2e^2} + \frac{1}{2}\left[-\frac{1}{2}e^{-2y}\right]_0^1$$

$$= -\frac{1}{2e^2} - \frac{1}{4}\left(\frac{1}{e^2} - 1\right) = -\frac{2}{4e^2} - \frac{1}{4e^2} + \frac{1}{4}$$

$$= \frac{1}{4} - \frac{3}{4e^2} = \boxed{\frac{1}{4}\left(1 - \frac{3}{e^2}\right)}.$$