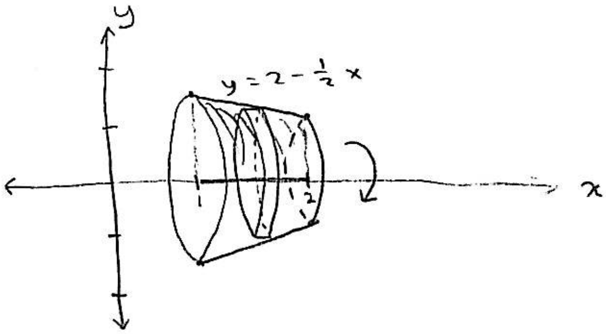
MATH 2 SOLUTIONS TO PROBLEM SET #10

SECTION 6.2 STEWART

(1.) $y = 2 - \frac{1}{2}x$, $y = 0$, $x = 1$, $x = 2$; ABOUT THE x-AXIS.



$$V = \pi \int_1^2 \left(2 - \tfrac{1}{2}x\right)^2 \, dx$$

$$= \pi \int_1^2 \left(\tfrac{1}{4}x^2 - 2x + 4\right) dx$$

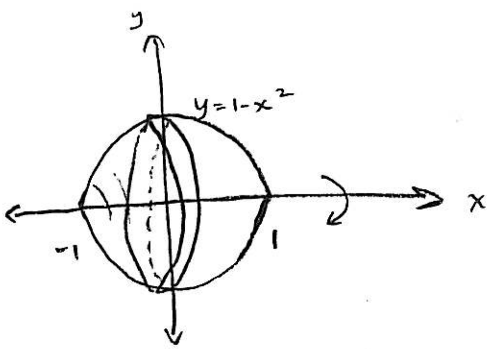
$$= \pi \left[\tfrac{1}{12}x^3 - x^2 + 4x\right]_1^2$$

$$= \pi \left[\left(\tfrac{8}{12} + 4\right) - \left(\tfrac{1}{12} + 3\right)\right]$$

$$= \boxed{\frac{19\pi}{12}}.$$

$\boxed{(2.)}$ $y = 1 - x^2$, $y = 0$; ABOUT THE x-AXIS.

$1 - x^2 = 0 \iff x^2 = 1 \iff x = 1$ OR -1, SO THE POINTS OF INTERSECTION OF THE PARABOLA AND THE LINE $y = 0$ (x-AXIS) ARE $(-1, 0)$ AND $(1, 0)$.



$$V = \pi \int_{-1}^{1} (1 - x^2)^2 \, dx$$

$$= \pi \int_{-1}^{1} (x^4 - 2x^2 + 1) \, dx$$

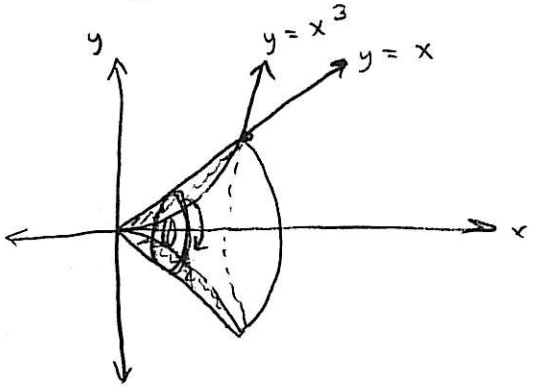
$$= \pi \left[\tfrac{1}{5}x^5 - \tfrac{2}{3}x^3 + x\right]_{-1}^{1}$$

$$= \pi\left(\left(\tfrac{1}{5} - \tfrac{2}{3} + 1\right) - \left(\tfrac{-1}{5} + \tfrac{2}{3} - 1\right)\right)$$

$$= \pi\left(\tfrac{2}{5} - \tfrac{4}{3} + 2\right)$$

$$= \pi\left(\tfrac{16}{15}\right) = \boxed{\frac{16\pi}{15}}.$$

(7.) $y = x^3$, $y = x$, $x \geq 0$; ABOUT THE X-AXIS



$x^3 = x \iff x^3 - x = 0$

$\iff x(x^2-1) = 0 \iff$

$x(x+1)(x-1) = 0$

$\iff x = 0, -1, \text{ OR } 1.$

$x \geq 0$ BY ASSUMTION, SO

$x = 0$ OR $x = 1$.

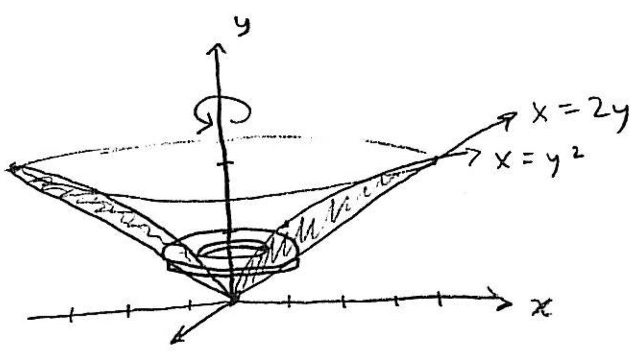
THUS THE GRAPHS OF $y = x^3$ AND $y = x$ $(x \geq 0)$ INTERSECT AT $(0,0)$ AND $(1,1)$.

$$V = \pi \int_0^1 x^2 - (x^3)^2 \, dx = \pi \int_0^1 x^2 - x^6 \, dx$$

$$= \pi \left[\tfrac{1}{3}x^3 - \tfrac{1}{7}x^7 \right]_0^1 = \pi\left(\tfrac{1}{3} - \tfrac{1}{7}\right) = \boxed{\frac{4\pi}{21}}.$$

(9.) $y^2 = x$, $x = 2y$; ABOUT THE y-AXIS

$y^2 = 2y \iff y^2 - 2y = 0 \iff y(y-2) = 0 \iff y = 0$ OR 2, SO THE GRAPHS INTERSECT AT $(0,0)$ AND $(4,2)$.



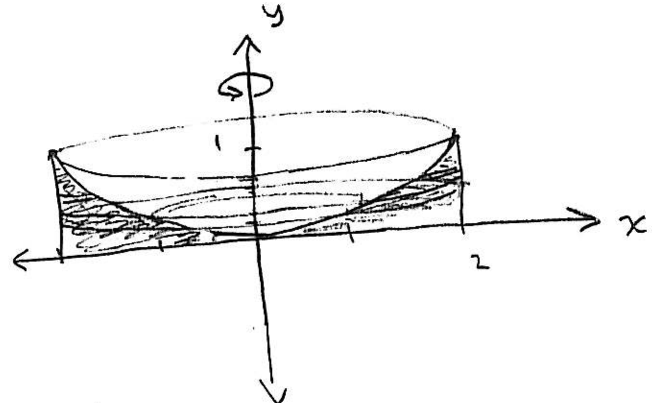
$$A = \pi \int_0^2 (2y)^2 - (y^2)^2 \, dy$$

$$= \pi \int_0^2 4y^2 - y^4 \, dy$$

$$= \pi \left[\tfrac{4}{3}y^3 - \tfrac{1}{5}y^5 \right]_0^2$$

$$= \pi\left(\tfrac{32}{3} - \tfrac{32}{5}\right) = \boxed{\frac{64\pi}{15}}.$$

$\boxed{(10.)}$ $y = \frac{1}{4}x^2$, $x = 2$, $y = 0$; ABOUT THE y-AXIS



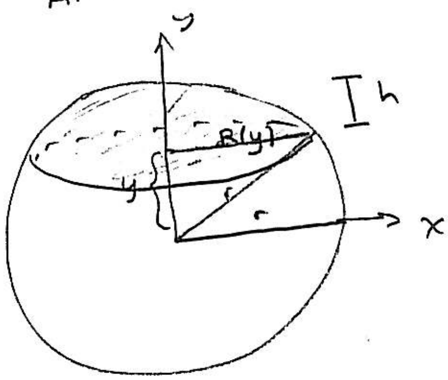
$y = \frac{1}{4}x^2 \quad (x \geq 0)$

$\Rightarrow x = 2\sqrt{y}$

$$V = \pi \int_0^1 2^2 - (2\sqrt{y})^2 \, dy = \pi \int_0^1 (4 - 4y)\, dy$$

$$= 4\pi \int_0^1 (1-y)\, dy = 4\pi \left[y - \frac{y^2}{2} \right]_0^1 = \boxed{2\pi}.$$

$\boxed{(51.)}$ FIND THE VOLUME OF A CAP OF A SPHERE WITH RADIUS r AND HEIGHT h.



THIS IS THE SOLID OF REVOLUTION OBTAINED BY REVOLVING THE REGION DEFINED BY $x = 0$, $x = R(y)$, $y = r-h$, $y = r$ ABOUT THE y-AXIS.

NOW, $y^2 + R(y)^2 = r^2$ BY THE PYTHAGOREAN THEOREM, SO $R(y) = \sqrt{r^2 - y^2}$.

$$\text{THUS} \quad V = \pi \int_{r-h}^{r} (r^2 - y^2)\, dy = \pi \left[r^2 y - \frac{y^3}{3} \right]_{r-h}^{r}$$

$$= \pi \left[\left(\frac{2}{3} r^3 \right) - \left(r^2(r-h) - \frac{(r-h)^3}{3} \right) \right]$$

$$= \pi \left(r^2 h + \frac{-3r^2h + 3rh^2 - h^3}{3} \right) = \pi \left(rh^2 - \frac{h^3}{3} \right)$$

$$= \boxed{\pi h^2 \left(r - \frac{h}{3} \right)}.$$

$\boxed{\underline{\text{NOTE}}: \text{ IF } h = 2r, \text{ THEN } V = \frac{4}{3}\pi r^3 \text{ (WHOLE SPHERE).}}$