Practice Exam 2

This is roughly double in length to the actual exam.

1. Find the area of the region in the plane bounded by the graphs of $x = y^3$ and $x = y^5$.

2. Find the area of the region in the plane bounded by the graphs of $y = \cos x, y = \sin x, x = 0$, and $x = \pi$. [Hint: To find any points of intersection in this interval, we must solve $\cos x = \sin x$. To solve this, use the Pythagorean angle identity: $\cos^2 x + \sin^2 x = 1$ for all x, or that $\tan x = \frac{\sin x}{\cos x}$.]

3. Find the area of the region in the plane bounded by the graphs of $x = y^2 - 4y + 3$ and $x = 3$.

4. Find the average value of $g(x) = x\cos(x^2)$ on $[0, \sqrt{\frac{\pi}{2}}]$.

5. Find the average value of $f(x) = \sec x \tan x$ on $[-\frac{\pi}{3}, \frac{\pi}{6}]$.

6. (a) Find the average value of $h(x) = \frac{1}{x^2}$ on $[1, 2]$.
   (b) Is there at least one number c, $1 < c < 2$, such that $h(c) = h_{ave}$? Why or why not? If so, find all such numbers c.

7. (a) Your instructor also works at a 30 foot climbing wall. Suppose an 80 pound kid cant do the climb and requests instead to be hauled at a constant speed all the way to the top. Assuming that the rope is of negligible weight and that theres no other forces involved (e.g. due to rope friction) how much work is done, in foot-pounds?
   (b) Suppose a bucket weighs 5 pounds when empty. A thirsty Vermont farmer on top of a 50 foot river bank has lowered the bucket and filled it with 40 pounds of water. He pulls it up at a constant speed, using a rope which weighs 1 pound per every 10 feet, but sadly, the bucket leaks 0.5 pounds of water per foot, so that it only has 15 pounds of water left when it reaches the top. Assuming there are no other forces involved (e.g. due to rope friction) how much work is done by the farmer, in foot-pounds?

8. A spring has a natural length of 50 centimeters. It takes a force of 28N to hold it stretched at 64 centimeters. How much work is done in stretching the spring from 60 centimeters to 80 centimeters? Include the units. [Hint: To get an answer in Newton-meters, i.e. Joules, convert all lengths to meters. There are 100 centimeters in a meter. Remember Hookes Law: F(x)=kx where k is a constant and x is the distance past the natural length.]

9. (a) What force is required to accelerate 30 kilos (kilograms) of stolen math goods across the Canadian border at 1.5 m/s^2? Include the units.

(b) Suppose this goes on for a distance of 50 meters, in order to evade the authorities, at which point you maintain your speed. How much extra work is done by your pickup truck in moving these weapons of math instruction across that 50 meter distance? Include the units.

10. Find the volume of the solid of revolution obtained by revolving the square region in the plane bounded by $x = 1, x = 3, y = 5$, and $y = 7$, about the x-axis. Use three methods, arriving at the same answer,

    (a) Use washers.

    (b) Use shells.

    (c) Use the formula for the volume of a cylinder.

11. Now find the volume of the solid of revolution obtained by revolving the same region in the previous problem, except about the y-axis. Use three methods, arriving at the same answer, which is, however, smaller than the answer to problem #10.

    (a) Use washers.

    (b) Use shells.

    (c) Use the formula for the volume of a cylinder.

12. Explain why the volume in #10 is smaller than the volume in #11.

13. Find the volume of the solid of revolution obtained by revolving the region in the plane bounded by $y = 0, y = e^{-x^2}, x = 0$, and $x = 1$, about the y-axis.

14. Find the volume of a solid (right circular) cone of height h and radius r. [Hint: Use disks. This is the solid of revolution obtained by revolving the region in the plane bounded by $y = 0, y = (r/h)x, x = 0$, and $x = h$; about the x-axis.]

15. Find the volume of the solid of revolution obtained by rotating the region in the plane bounded by $x = 0, x = \frac{1}{\sqrt{y}}, y = 3$, and $y = 5$; about the y-axis.

16. Find the volume of the solid of revolution obtained by revolving the region in the plane bounded by $x = y\cos(y^3), x = ye^{y^3}, y = 0$, and $y = \pi^{\frac{1}{3}}$, about the x-axis.

Practice Bonus :

(a) For each of problems 13,15,16, what method did you use? Why?

(b) You apply a force of $100(x^2 + \sin x)$ pounds at x feet, in pushing a stubborn moose, whose cooperation is apparently variable. How much work is done in pushing the moose from 0 to π feet? Include the units.