

$$11. (f) \int \frac{1-\tan\theta}{1+\tan\theta} d\theta = \int \frac{1-\tan\theta}{1+\tan\theta} \cdot \frac{1-\tan\theta}{1-\tan\theta} d\theta$$

$$= \int \frac{1-2\tan\theta + \tan^2\theta}{1-\tan^2\theta} d\theta = \int \frac{\sec^2\theta - 2\tan\theta}{1-\tan^2\theta} d\theta$$

$$= \int \frac{\frac{1}{\cos^2\theta} - \frac{2\sin\theta}{\cos\theta}}{1 - \frac{\sin^2\theta}{\cos^2\theta}} d\theta = \int \frac{\frac{1-2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}} d\theta$$

$$= \int \frac{1-2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} d\theta = \int \frac{1-\sin 2\theta}{\cos 2\theta} d\theta = \int \sec(2\theta) - \tan(2\theta) d\theta$$

$$u=2\theta \quad du=2d\theta$$

$$= \frac{1}{2} \int \sec u - \tan u \, du = \frac{1}{2} \left[ \ln|\sec u + \tan u| - \ln|\sec u| \right] + C$$

$$= \frac{1}{2} \ln \left| \frac{\sec u + \tan u}{\sec u} \right| + C = \frac{1}{2} \ln \left| \frac{1 + \sin u}{1} \right| + C$$

$$= \frac{1}{2} \ln |1 + \sin(2\theta)| + C$$