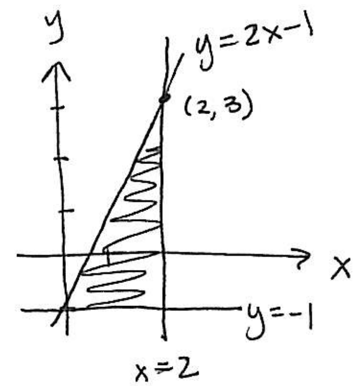
FINAL REVIEW - SOLUTIONS



1. $y = 2x - 1$, $y = -1$, $x = 2$

$y + 1 = 2x$

$\frac{y+1}{2} = x$

a) AREA

i) geometric formula

$$A = \tfrac{1}{2} bh = \tfrac{1}{2}(2)(4) = 4$$

ii) limit of a Riemann Sum

rectangle: width $= \Delta x = \frac{2}{n}$ $\qquad x_i = 0 + i\Delta x = \frac{2i}{n}$

height $= f(x_i) - (-1)$

$= 2\left(\frac{2i}{n}\right) - 1 + 1 = \frac{4i}{n}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4i}{n} \cdot \frac{2}{n} = \lim_{n\to\infty} \frac{8}{n^2} \sum_{i=1}^{n} i = \lim_{n\to\infty} \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= 4 \lim_{n\to\infty} \frac{n^2+n}{n^2} = 4$$

iii) integral

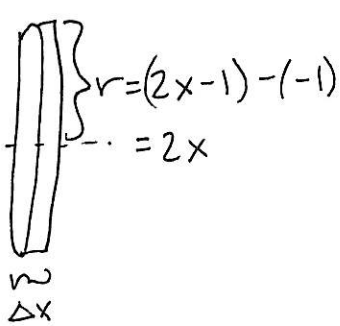
$$A = \int_0^2 (2x-1) - (-1)\,dx = \int_0^2 2x\,dx = x^2 \Big|_0^2 = 4$$

b) VOLUME - about $y = -1$

i) geometric formula

CONE: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (4)^2(2) = \frac{32\pi}{3}$

ii) slicing $\longrightarrow$ disks

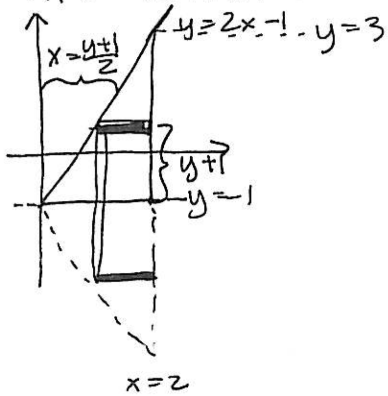


$V_{disk} = \pi r^2 \Delta x = \pi (2x)^2 \Delta x = 4\pi x^2 \Delta x$

$$V = \int_0^2 4\pi x^2\,dx = 4\pi \frac{x^3}{3} \Big|_0^2 = 4\pi\left(\frac{8}{3} - 0\right)$$

$$= \frac{32\pi}{3}$$

iii) Volume - shells



$$V_{shell} = 2\pi r h \Delta y = 2\pi(y+1)\left(2-\frac{y+1}{2}\right)\Delta y$$
$$= 2\pi(y+1)\left(\frac{3}{2}-\frac{y}{2}\right)\Delta y$$
$$= \pi(y+1)(3-y)\Delta y = \pi(3+2y-y^2)\Delta y$$

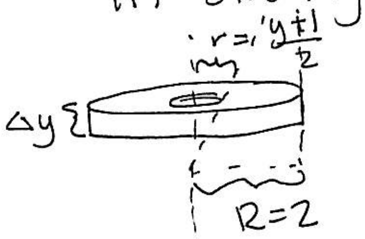
$$V = \int_{-1}^{3}\pi(3+2y-y^2)\,dy = \pi\left[3y+y^2-\frac{y^3}{3}\right]_{-1}^{3}$$
$$= \pi\left[\left(9+9-\frac{27}{3}\right)-\left(-3+1+\frac{1}{3}\right)\right] = \pi\left(9+2-\frac{1}{3}\right)$$
$$= \frac{32\pi}{3}$$

c) VOLUME - about y-axis

i) geometric formulas

$$(\text{CYLINDER})-(\text{CONE}) = \pi r^2 h - \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^2 h$$
$$= \frac{2}{3}\pi(2)^2(4) = \frac{32\pi}{3}$$

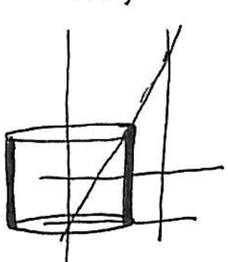
ii) slicing $\rightarrow$ washers



$$V_{washer} = \pi(R^2-r^2)\Delta y = \pi\left(2^2-\left(\frac{y+1}{2}\right)^2\right)\Delta y$$
$$= \pi\left(4-\frac{y^2+2y+1}{4}\right)\Delta y = \frac{\pi}{4}(16-y^2-2y-1)\Delta y$$
$$= \frac{\pi}{4}(15-y^2-2y)\Delta y$$

$$V = \int_{-1}^{3}\frac{\pi}{4}(15-y^2-2y)\,dy = \frac{\pi}{4}\left[15y-\frac{y^3}{3}-y^2\right]_{-1}^{3} = \frac{\pi}{4}\left[\left(45-\frac{27}{3}-9\right)-\left(-15+\frac{1}{3}-1\right)\right]$$
$$= \frac{\pi}{4}\left(45-9-9+15-\frac{1}{3}+1\right) = \frac{\pi}{4}\left(43-\frac{1}{3}\right) = \frac{\pi}{4}\left(\frac{129-1}{3}\right) = \frac{32\pi}{3}$$

iii) shells



$$V_{shell} = 2\pi r h \Delta x = 2\pi(x)(2x-1-(-1))\Delta x = 4\pi x^2\Delta x$$
$$V = \int_0^2 4\pi x^2\,dx = 4\pi\frac{x^3}{3}\Big|_0^2 = 4\pi\left(\frac{8}{3}-0\right) = \frac{32\pi}{3}$$

2. Find the derivative ... FToC Part I

a) $f(x) = \int_a^b g(t)\,dt \leftarrow$ a constant

$f'(x) = 0$

b) $F(x) = \int_x^1 \sqrt{t+\sin t}\,dt = -\int_1^x \sqrt{t+\sin t}\,dt$

$F'(x) = -\sqrt{x+\sin x}$

c) $G(x) = \int_0^x \frac{t^2}{1+t^3}\,dt \longrightarrow G'(x) = \frac{x^2}{1+x^3}$

d) $y = \int_{\sqrt{x}}^x \frac{e^t}{t}\,dt = \int_{\sqrt{x}}^0 \frac{e^t}{t}\,dt + \int_0^x \frac{e^t}{t}\,dt = -\int_0^{\sqrt{x}} \frac{e^t}{t}\,dt + \int_0^x \frac{e^t}{t}\,dt$

$y' = \left(-\frac{e^{\sqrt{x}}}{\sqrt{x}}\right)\left(\sqrt{x}\right)' + \frac{e^x}{x} = \left(-\frac{e^{\sqrt{x}}}{\sqrt{x}}\right)\left(\frac{1}{2\sqrt{x}}\right) + \frac{e^x}{x} = \frac{-e^{\sqrt{x}}}{2x} + \frac{e^x}{x}$

chain rule

$= \frac{2e^x - e^{\sqrt{x}}}{2x}$

3. $v(t) = t^2 - t$

a) DISPLACEMENT $= \int_0^5 t^2 - t\,dt = \frac{t^3}{3} - \frac{t^2}{2}\Big|_0^5 = \frac{125}{3} - \frac{25}{2} = \frac{175}{6}$ m

b) $t^2 - t = 0$

$t(t-1) = 0$

$t = 0, 1$

$v(2) = 4 - 2 = 2 > 0$

$v(t) \geq 0$ for $t \geq 1$

$v(t) \leq 0$ for $0 \leq t \leq 1$

DIST $= \int_0^1 -t^2 + t\,dt + \int_1^5 t^2 - t\,dt$

$= \left[-\frac{t^3}{3} + \frac{t^2}{2}\right]_0^1 + \left[\frac{t^3}{3} - \frac{t^2}{2}\right]_1^5 = \left[-\frac{1}{3} + \frac{1}{2} - 0\right] + \left[\frac{125}{3} - \frac{25}{2} - \frac{1}{3} + \frac{1}{2}\right]$

$= \left(\frac{1}{6}\right) + \left(\frac{124}{3} - \frac{24}{2}\right) = \frac{1 + 248 - 72}{6} = \frac{177}{6} = \frac{59}{2}$ m

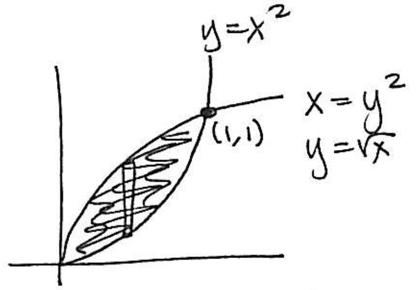
4. a) $\int_0^1 \frac{d}{dx}\left(e^{\arctan x}\right) dx$ antiderivative of the derivative of f is f FToC II

$= e^{\arctan(1)} - e^{\arctan(0)} = e^{\pi/4} - e^{0} = e^{\pi/4} - 1$

b) $\frac{d}{dx} \underbrace{\int_0^1 e^{\arctan x}\, dx}_{\text{constant}} = 0$

c) $\frac{d}{dx} \int_0^x e^{\arctan t}\, dt = e^{\arctan x}$ FToC Part I

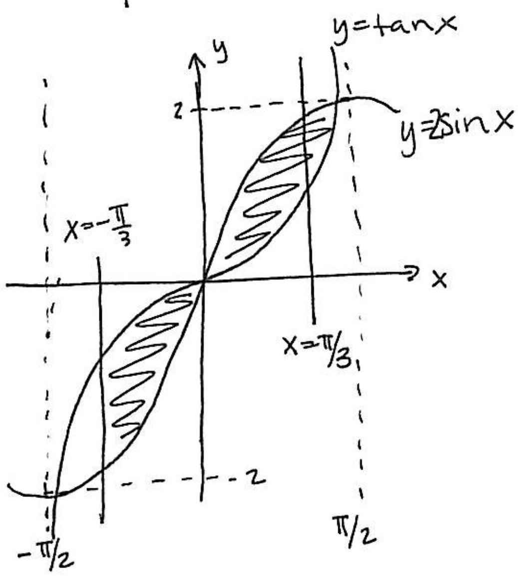
5. a)



$A = \int_0^1 \sqrt{x} - x^2\, dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{3/2} - \frac{1}{3} - 0 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

b)



$A = \int_{-\pi/3}^{0} \tan x - \qquad dx$

$+ \int_0^{\pi/3} 2\sin x - \tan x\, dx$

BUT BY SYMMETRY...

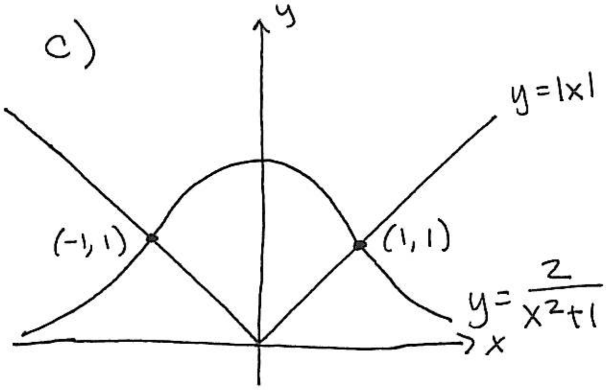
$= 2\int_0^{\pi/3} 2\sin x - \tan x\, dx$

$= 2\left[-2\cos x - \ln|\sec x|\right]_0^{\pi/3}$

$= 2\left[\left(-2\cos(\pi/3) - \ln\left|\frac{1}{\cos(\pi/3)}\right|\right) - \left(-2\cos(0) - \ln\left|\frac{1}{\cos(0)}\right|\right)\right]$

$= 2\left[-2\left(\frac{1}{2}\right) - \ln\left|\frac{1}{1/2}\right| + 2(1) + \cancel{\ln\left|\frac{1}{1}\right|}^{0}\right] = 2\left[1 - \ln 2\right] = 2 - \ln 2.$

c)



$$\frac{2}{x^2+1} = x$$

$$2 = x^3 + x$$

$$0 = x^3 + x - 2$$

$$0 = (x-1)(x^2+x+2)$$

$$x = 1$$

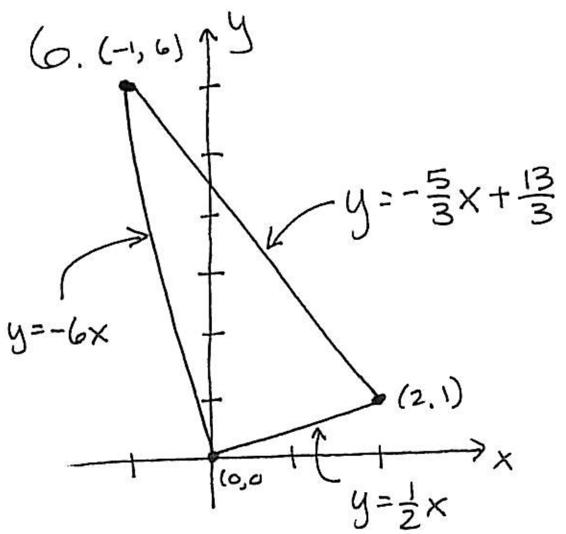
$$A = \int_{-1}^{0} \frac{2}{x^2+1} - (-x)\, dx + \int_0^1 \frac{2}{x^2+1} - x\, dx$$ ... OR AGAIN WITH SYMMETRY

$$= 2\int_0^1 \frac{2}{x^2+1} - x\, dx = 2\left[2\arctan x - \frac{x^2}{2}\right]_0^1$$

$$= 2\left(2\arctan(1) - \frac{1}{2} - 2\arctan(0) + 0\right)$$

$$= 2\left(2\left(\frac{\pi}{4}\right) - \frac{1}{2} - 0\right) = \pi - 1$$

6.



$$A = \int_{-1}^{0} \left(-\frac{5}{3}x + \frac{13}{3}\right) - (-6x)\, dx$$

$$+ \int_0^2 \left(-\frac{5}{3}x + \frac{13}{3}\right) - \left(\frac{1}{2}x\right) dx$$

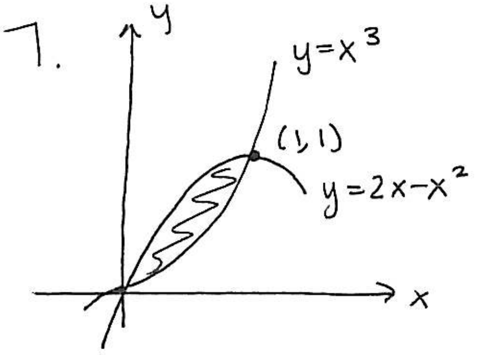
$$= \int_{-1}^{0} \frac{13}{3}x + \frac{13}{3} dx + \int_0^2 -\frac{13}{3}x + \frac{13}{3}\, dx$$

$$= \frac{13}{3}\left[\int_{-1}^{0} x + 1\, dx + \int_0^2 -x + 1\, dx\right]$$

$$= \frac{13}{3}\left[\left(\frac{x^2}{2} + x\right)\Big|_{-1}^{0} + \left(-\frac{x^2}{2} + x\right)\Big|_0^2\right]$$

$$= \frac{13}{3}\left[(0+0) - \left(\frac{1}{2} - 1\right) + \left(-\frac{4}{2} + 2\right) - (0+0)\right] = \frac{13}{3}\left[\frac{1}{2}\right] = \frac{13}{6}$$

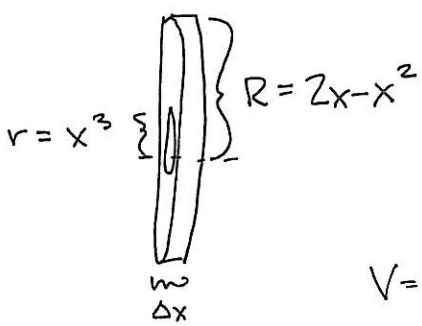
7.



$$x^3=2x-x^2$$
$$x^3+x^2-2x=0$$
$$x(x^2+x-2)=0$$
$$x(x+2)(x-1)=0$$
$$x=0,-2,1$$

a) $A=\int_0^1 (2x-x^2)-(x^3)\,dx = x^2-\frac{x^3}{3}-\frac{x^4}{4}\Big|_0^1 = 1-\frac{1}{3}-\frac{1}{4}=\frac{5}{12}$

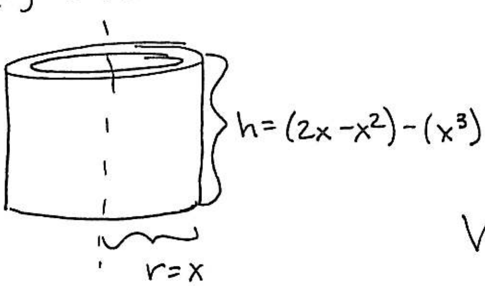
b) WASHERS



$$V_{washer}=\pi(R^2-r^2)\Delta x$$
$$=\pi\left((2x-x^2)^2-(x^3)^2\right)\Delta x$$
$$=\pi\left(4x^2-4x^3+x^4-x^6\right)\Delta x$$

$$V=\int_0^1 \pi\left(4x^2-4x^3+x^4-x^6\right)dx=\pi\left[\frac{4x^3}{3}-x^4+\frac{x^5}{5}-\frac{x^7}{7}\right]_0^1$$
$$=\pi\left(\frac{4}{3}-1+\frac{1}{5}-\frac{1}{7}\right)=\pi\left(\frac{140-105+21-35}{105}\right)=\pi\left(\frac{21}{105}\right)=\frac{\pi}{5}$$

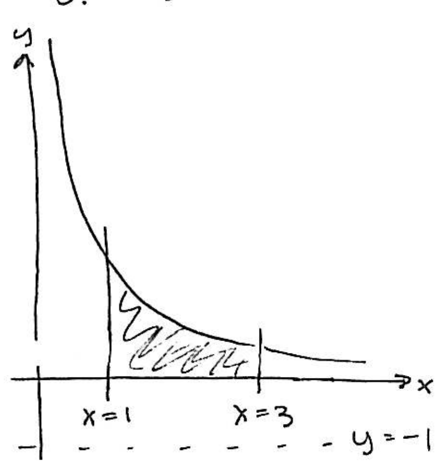
c) SHELLS



$$V_{shell}=2\pi rh\Delta x=2\pi x(2x-x^2-x^3)\Delta x$$
$$=2\pi\left(2x^2-x^3-x^4\right)\Delta x$$

$$V=\int_0^1 2\pi\left(2x^2-x^3-x^4\right)dx=2\pi\left[\frac{2x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}\right]_0^1$$
$$=2\pi\left(\frac{2}{3}-\frac{1}{4}-\frac{1}{5}-0\right)=2\pi\left(\frac{40-15-12}{60}\right)=\frac{13\pi}{30}$$

8. a)



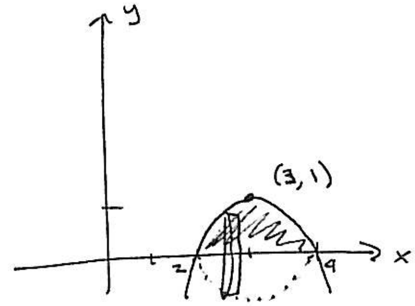
$$V_{washer}=\pi(R^2-r^2)\Delta x$$
$$=\pi\left(\left(\tfrac{1}{x}+1\right)^2-(1)^2\right)\Delta x=\pi\left(\frac{1}{x^2}+\frac{2}{x}\cancel{+1-1}\right)\Delta x$$
$$=\pi\left(x^{-2}+\frac{2}{x}\right)\Delta x$$

$$V=\int_1^3 \pi\left(x^{-2}+\frac{2}{x}\right)dx=\pi\left[\frac{x^{-1}}{-1}+2\ln|x|\right]_1^3$$
$$=\pi\left[\left(-\frac{1}{3}+2\ln 3\right)+\left(-1+2\cancel{\ln|1|}^{\,0}\right)\right]=\pi\left(2\ln 3-\frac{4}{3}\right)$$

b) $y = -x^2+6x-8$

$= -(x^2-6x+9)+9-8$

$= 1-(x-3)^2$



$V_{disk} = \pi r^2 \Delta x = \pi\left(1-(x-3)^2\right)^2 \Delta x$

$= \pi\left(1-(x^2-6x+9)\right)^2 \Delta x = \pi\left(-8-x^2+6x\right)^2 \Delta x$

$= \pi\left(64+8x^2-48x+8x^2+x^4-6x^3-48x-6x^3+36x^2\right)\Delta x$

$= \pi\left(64-96x+52x^2-12x^3+x^4\right)\Delta x$

$V = \int_2^4 \pi\left(64-96x+52x^2-12x^3+x^4\right)dx$

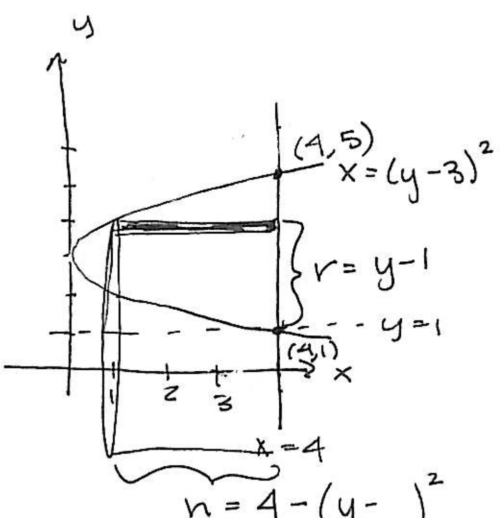
$= \pi\left[64x-48x^2+\frac{52x^3}{3}-3x^4+\frac{x^5}{5}\right]_2^4$

$= \pi\left[64(4-2)-48(16-4)+\frac{52}{3}(64-8)-3(256-16)+\frac{1}{5}(1024-32)\right]$

$= \pi\left[128-576+\frac{2912}{3}-720+\frac{992}{5}\right] = \pi\left[-1,168+\frac{2912}{3}+\frac{992}{5}\right]$

$= \pi\left[\frac{-17520+14560+2976}{15}\right] = -\frac{16}{15}\pi$

c)



$V_{shell} = 2\pi r h \Delta y$

$= 2\pi(y-1)\left(4-(y-1)^2\right)\Delta y$

$= 2\pi(y-1)\left(4-y^2+2y-1\right)\Delta y$

$= 2\pi(y-1)\left(-y^2+2y+3\right)\Delta y$

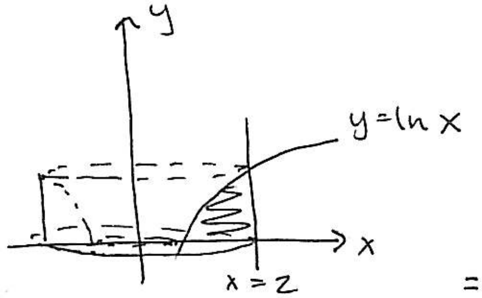
$= 2\pi\left(-y^3+3y^2+y-3\right)\Delta y$

$V = \int_1^5 2\pi\left(-y^3+3y^2+y-3\right)dy$

$= 2\pi\left[-\frac{y^4}{4}+y^3+\frac{y^2}{2}-3y\right]_1^5 = 2\pi\left[\left(-\frac{625}{4}+125+\frac{25}{2}-15\right)-\left(-\frac{1}{4}+1+\frac{1}{2}-3\right)\right]$

$= 2\pi\left(-31+126\cancel{+12-12}\right) = 190\pi$

d)



$V_{shell} = 2\pi r h \Delta x$

$= 2\pi(x)(\ln x)\Delta x$

$V = \int_1^2 2\ x \ln x\, dx$ $\quad u = x \quad dv = \ln x\, dx$

$\quad du = dx \quad v = ?$

$= x(x\ln x - x)\Big|_1^2 - \int_1^2 (x\ln x - x)dx$

$= x^2 \ln x - x^2\Big|_1^2 + \frac{x^2}{2}\Big|_1^2 - \int_1^2 x\ln x\, dx$

$\boxed{? = \int \ln x\, dx = x\ln x - \int x \cdot \frac{1}{x} dx = x\ln x - x + C}$ $= 4\ln 2 - 4 - \cancel{\ln 1} + 1 + \frac{4}{2} - \frac{1}{2} - \int_1^2 x\ln x\, dx$

$u = \ln x \quad du = \frac{1}{x}dx$ $\quad = 4\ln 2 - \frac{3}{2} - \int_1^2 x \ln x\, dx$

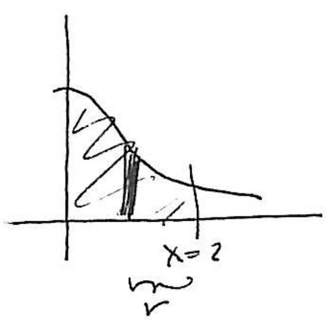
$dv = dx \quad v = x$

$2\pi \int_1^2 x \ln x\, dx = 4\ln 2 - \frac{3}{2} - \int_1^2 x \ln x\, dx$

$(2\pi + 1)\int_1^2 x \ln x\, dx = 4\ln 2 - \frac{3}{2}$

$V = \dfrac{4\ln 2 - \frac{3}{2}}{2\pi + 1}$

e)



$V_{shell} = 2\pi r h \Delta x = 2\pi(2-x)\left(\frac{1}{1+x^2}\right)\Delta x$

$= 2\pi\left(\frac{2-x}{1+x^2}\right)\Delta x$

$V = \int_0^2 2\pi\left(\frac{}{1+x^2} - \frac{x}{1+x^2}\right)dx$

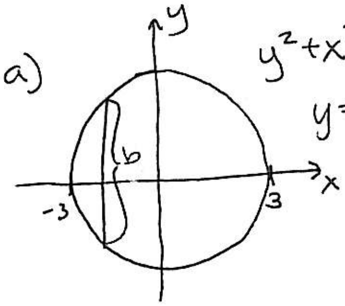
$= \pi \int_0^2 \frac{}{1+x^2}dx - 2\pi \int_0^2 \frac{x}{1+x^2}dx$ $\quad u = 1+x^2$, $du = 2x\, dx$, $0 \to 1 \quad 2 \to 5$

$= 4\pi \arctan x\Big|_0^2 - \pi \int_1^5 \frac{du}{u} = 4\pi(\arctan 2 - \text{artan}0) - \pi \ln u\Big|_1^5$

$= 4\pi(\tan^{-1} 2 - 0) - \pi(\ln 5 - \cancel{\ln 1}^{0}) = 4\pi \tan^{-1} 2 - \pi \ln 5$

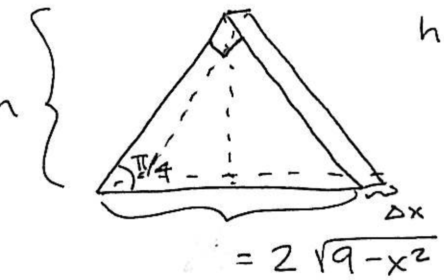
9.

a)



$y^2+x^2=9$

$y=\pm\sqrt{9-x^2}$



$h=\left(\sqrt{9-x^2}\right)\tan \pi/4$

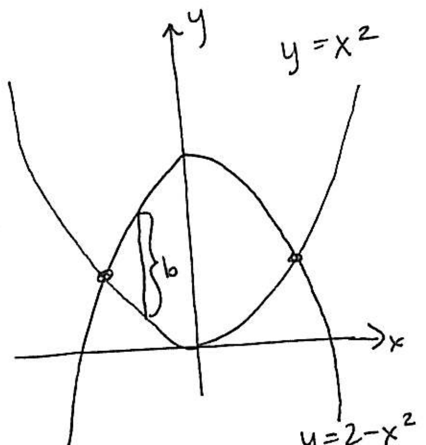
$= \sqrt{9-x^2}$

$$V_{slice}=\frac{1}{2}bh\Delta x=\frac{1}{2}\left(2\sqrt{9-x^2}\right)\left(\sqrt{9-x^2}\right)\Delta x=\left(9-x^2\right)\Delta x$$

$$V=\int_{-3}^{3} 9-x^2\,dx = 9x-\frac{x^3}{3}\Big|_{-3}^{3}=27-\frac{27}{3}-(-27)+\frac{-27}{3}$$

$$=54-\frac{54}{3}=54-18=36$$

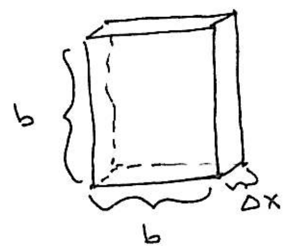
b)



$x^2=2-x^2$

$2x^2=2$

$x=\pm 1$



$b=(2-x^2)-(x^2)$

$=2-2x^2$

$$V_{slice}=b^2\Delta x=(2-2x^2)^2\Delta x$$

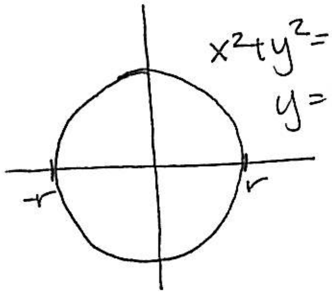
$$=(4-8x^2+4x^4)\Delta x$$

$$V=\int_{-1}^{1} 4-8x^2+4x^4\,dx = 4x-\frac{8}{3}x^3+\frac{4}{5}x^5\Big|_{-1}^{1}$$

$$=\left(4-\frac{8}{3}+\frac{4}{5}\right)-\left(-4+\frac{8}{3}-\frac{4}{5}\right)=8-\frac{16}{3}+\frac{8}{5}=\frac{120-80-24}{15}$$

$$=\frac{16}{15}$$

10. a)



$x^2+y^2=r^2$

$y=\pm\sqrt{r^2-x^2}$

$A=2\int_{-r}^{r}\sqrt{r^2-x^2}\,dx$

$x=r\sin\theta \qquad \sqrt{r^2-x^2}=r\sqrt{\cos^2\theta}$

$dx=r\cos\theta\, d\theta \qquad =r\cos\theta$

$x=-r \longrightarrow \theta=-\pi/2 \qquad x=r\longrightarrow \theta=\pi/2$

$$=2\int_{-\pi/2}^{\pi/2}(r\cos\theta)(r\cos\theta)\,d\theta = 2r^2\int_{-\pi/2}^{\pi/2}\cos^2\theta\, d\theta$$

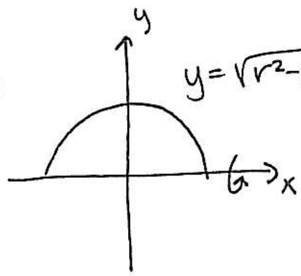
$$=2r^2\int_{-\pi/2}^{\pi/2}\frac{1+\cos 2\theta}{2}\,d\theta=\frac{1}{2}r^2\int_{-\pi}^{\pi}1+\cos u\;du$$

$u=2\theta \qquad -\pi/2\to-\pi$

$du=2d\theta \qquad \pi/2\to\pi$

$$=\frac{1}{2}r^2\Big[u+\sin u\Big]_{-\pi}^{\pi}=\frac{r^2}{2}\big(\pi+0-(-\pi)-0\big)=\pi r^2 \checkmark$$

b)



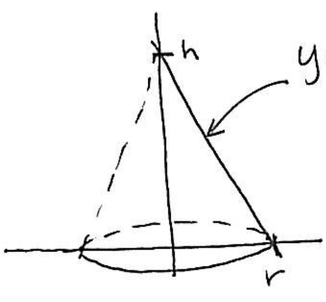
DISK

$V_{disk}=\pi\left(\sqrt{r^2-x^2}\right)^2\Delta x=\pi(r^2-x^2)\Delta x$

$V=\int_{-r}^{r}\pi(r^2-x^2)\,dx=\pi\left[r^2x-\frac{x^3}{3}\right]_{-r}^{r}$

$$=\pi\left[\left(r^2\cdot r-\frac{r^3}{3}\right)-\left(r^2\cdot -r-\frac{-r^3}{3}\right)\right]=\pi\left(2r^3-\frac{2r^3}{3}\right)=\frac{4}{3}\pi r^3 \checkmark$$

c)



$y=-\frac{h}{r}x+h$

SHELLS

$V_{shell}=2\pi(x)\left(-\frac{h}{r}x+h\right)\Delta x$

$=2\pi\left(-\frac{h}{r}x^2+hx\right)\Delta x$

$$V=\int_0^r 2\pi\left(-\frac{h}{r}x^2+hx\right)dx=2\pi\left[-\frac{hx^3}{3r}+\frac{hx^2}{2}\right]_0^r$$

$$=2\pi\left[-\frac{hr^3}{3r}+\frac{hr^2}{2}-0\right]=\cancel{2}\pi\left(\frac{-2hr^3+3hr^3}{3\cancel{6}r}\right)$$

$=\frac{1}{3}\pi r^2 h \checkmark$

11. a) $\int_1^4 \frac{dt}{(2x+1)^3} = \frac{1}{(2x+1)^3}\int_1^4 dt = \frac{1}{(2x+1)^3}\, t\Big|_1^4 = \frac{3}{(2x+1)^3}$

THE WAY IT IS WRITTEN

WITHOUT THE TYPO

$\int_1^4 \frac{dt}{(2t+1)^3} = \frac{1}{2}\int_3^9 \frac{1}{u^3}\,du = \frac{1}{2}\,\frac{u^{-2}}{-2}\Big|_3^9 = -\frac{1}{4}\left(\frac{1}{81}-\frac{1}{9}\right)$

$u = 2t+1 \qquad 1 \to 3$ $\qquad = \frac{2}{81}$

$du = 2dt \qquad 4 \to 9$

b) $\int_0^1 \frac{\sqrt{\arctan x}}{x^2+1}\,dx = \int_0^{\pi/4} \sqrt{u}\,du = \frac{u^{1/2}}{1/2}\Big|_0^{\pi/4} = 2\left(\sqrt{\pi/4} - \sqrt{0}\right)$

$u = \arctan x \qquad 0 \to 0$ $\qquad = 2\sqrt{\frac{\pi}{4}} = \sqrt{\pi}$

$du = \frac{dx}{x^2+1} \qquad 1 \to \pi/4$

c) $\int \frac{1}{y^2-4y-12}\,dy = \int \frac{1/8}{y-6} + \frac{-1/8}{y+2}\,dy = \frac{1}{8}\left(\ln|y-6| - \ln|y+2|\right) + C$

$\frac{A}{y-6} + \frac{B}{y+2} = \frac{1}{y^2-4y-12}$ $\qquad = \frac{1}{8}\ln\left|\frac{y-6}{y+2}\right| + C$

$A(y+2) + B(y-6) = 1$

$(A+B)y + 2A - 6B = 1$

$A+B=0 \qquad 2A-6B=1$

$A=-B \qquad -2B-6B=1$

$B = -\frac{1}{8}$

$A = \frac{1}{8}$

d) $\int \frac{\sec^6\theta}{\tan^2\theta}\,d\theta = \int \frac{\sec^4\theta\,\sec^2\theta}{\tan^2\theta}\,d\theta = \int \frac{(1+\tan^2\theta)^2\sec^2\theta}{\tan^2\theta}\,d\theta$

$u = \tan\theta$
$du = \sec^2\theta\,d\theta$

$= \int \frac{(1+u^2)^2}{u^2}\,du = \int \frac{1+2u^2+u^4}{u^2}\,du$

$= \int u^{-2}+2+u^2\,du = \frac{u^{-1}}{-1}+2u+\frac{u^3}{3}+C = \frac{u^4+6u^2-3}{3u}+C$

e) $\int t e^{\sqrt{t}}\,dt = \int w^2 e^w\,2w\,dw = 2\int w^3 e^w\,dw$

$w=\sqrt{t}$ $\quad w^2 = t$

$2w\,dw = dt$

$u = w^3$ $\quad dv = e^w\,dw$

$du = 3w^2$ $\quad v = e^w$

$= 2\left[w^3e^w - \int 3w^2e^w\,dw\right] = 2\left[w^3e^w - 3w^2e^w + \int 6we^w\,dw\right]$

$= 2\left[w^3e^w - 3w^2e^w + 6we^w - \int 6e^w\,dw\right] = 2w^3e^w - 6w^2e^w + 12we^w - 12e^w + C$

$\int 3w^2e^w\,dw = 3w^2e^w - \int 6we^w\,dw$

$u = 3w^2$ $\quad dv = e^w\,dw$

$du = 6w\,dw$ $\quad v = e^w$

$\int 6we^w\,dw = 6we^w - \int 6e^w\,dw$

$u = 6w$ $\quad dv = e^w\,dw$

$du = 6\,dw$ $\quad v = e^w$

$= 2t^{3/2}e^{\sqrt{t}} - 6te^{\sqrt{t}} + 12\sqrt{t}\,e^{\sqrt{t}} - 12e^{\sqrt{t}} + C$

f) $\int \frac{1-\tan\theta}{1+\tan\theta}\, d\theta = \int \frac{(1-\tan\theta)}{(1+\tan\theta)} \cdot \frac{(1-\tan\theta)}{(1-\tan\theta)}\, d\theta$

$$= \int \frac{1-2\tan\theta+\tan^2\theta}{\underbrace{1-\tan^2\theta}_{\sec^2\theta}}\, d\theta = \int \cos^2\theta - \frac{2\tan\theta}{\sec^2\theta} + \frac{(1-\sec^2\theta)}{\sec^2\theta}\, d\theta$$

$$= \int 2\cos^2\theta - 2\sin\theta\cos\theta - 1\, d\theta$$

$$= \int \cancel{1}+\cos 2\theta - \sin 2\theta \cancel{-1}\, d\theta \qquad \begin{aligned} 2\theta &= u \\ du &= 2d\theta \end{aligned}$$

$$= \frac{1}{2}\int \cos u - \sin u\, du = \frac{1}{2}(\sin u + \cos u) + C$$

$$= \frac{1}{2}(\sin 2\theta + \cos 2\theta) + C$$

g) $\int_0^{\pi/4} \tan^5\theta \sec^3\theta\, d\theta = \int_0^{\pi/4} \tan^4\theta \sec^2\theta (\sec\theta\tan\theta) d\theta$

$$= \int_0^{\pi/4} (\sec^2\theta - 1)^2 \sec^2\theta\ (\sec\theta\tan\theta) d\theta$$

$u = \sec\theta \qquad du = \sec\theta\tan\theta\, d\theta$

$0 \to 1 \qquad \pi/4 \to \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$$= \int_1^{\sqrt{2}} (u^2-1)^2 u^2\, du = \int_1^{\sqrt{2}} (u^4-2u^2+1)u^2\, du$$

$$= \int_1^{\sqrt{2}} u^6 - 2u^4 + u^2\, du = \left.\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right|_1^{\sqrt{2}}$$

$$= \left(\frac{\sqrt{2}^7}{7} - \frac{2\sqrt{2}^5}{5} + \frac{\sqrt{2}^3}{3}\right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right)$$

$$= \frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}$$

h) $\int x^2 \sin x\, dx$

$u = x^2 \qquad dv = \sin x\, dx$

$du = 2x\,dx \qquad v = -\cos x$

$= -x^2\cos x + \int 2x\cos x\, dx$

$p = 2x \qquad dq = \cos x\, dx$

$dp = 2dx \qquad q = \sin x$

$= -x^2\cos x + 2x\sin x - \int 2\sin x\, dx$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

12. For springs $F(x) = kx$.

$10\text{cm} = .1\text{ m}$

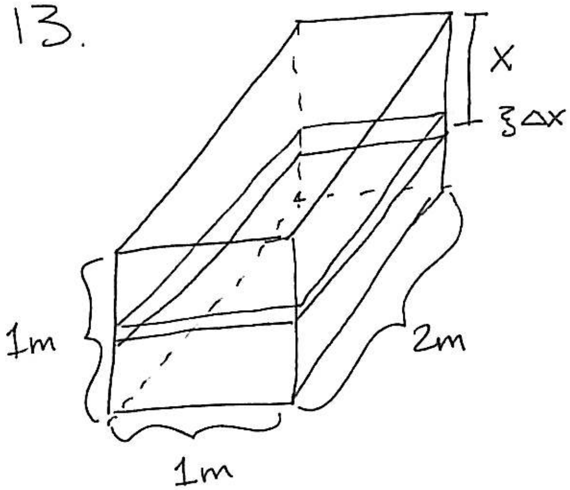
$W = \int F(x)dx$

$W_1 = \int_0^{.1} kx\,dx = \frac{k}{2}x^2\Big|_0^{.1} = \frac{k}{2}(.1^2 - 0) = \frac{k}{2}(.01) = \frac{k}{200}$

$W_2 = \int_{.1}^{.2} kx\,dx = \frac{k}{2}x^2\Big|_{.1}^{.2} = \frac{k}{2}(.2^2 - .1^2) = \frac{k}{2}\left(\frac{1}{25} - \frac{1}{100}\right)$

$= \frac{3k}{200} = 3(W_1)$

13.



$$V_{slab} = (2)(1)(\Delta x)$$
$$= 2\Delta x$$

$$m_{slab} = V_{slab} \cdot 1000$$
$$= 2000\,\Delta x$$

$$F_{slab} = m_{slab} \cdot a$$
$$= (2000\Delta x)(9.8)$$
$$= 19600\,\Delta x$$

$$W_{slab} = F_{slab} \cdot x = 19600\,x\,\Delta x$$

$$W = \int_0^{.5} 19600x\,dx = 9800x^2\Big|_0^{.5} = 9800\left(\tfrac{1}{4} - 0\right)$$

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14. a) $f(x) = \sqrt{x}$ on $[0,4]$.

$$f_{ave} = \frac{1}{4-0}\int_0^4 x^{1/2}\,dx = \frac{1}{4}\,\frac{x^{3/2}}{3/2}\Big|_0^4 = \frac{1}{6}(4)^{3/2} = \frac{4}{3}$$

$$f(c) = f_{ave}$$
$$\sqrt{c} = \frac{4}{3}$$
$$c = \frac{16}{9}$$

b) $g(x) = 2 + 6x - 3x^2$ on $[0, 2]$

$$g_{ave} = \frac{1}{2-0}\int_0^2 2 + 6x - 3x^2\,dx = \frac{1}{2}\left[2x + 3x^2 - x^3\right]_0^2$$

$$= \frac{1}{2}\left[4 + 12 - 8 - 0\right] = 4$$

$g(c) = g_{ave}$

$2 + 6c - 3c^2 = 4$

$3c^2 - 6c + 2 = 0$

$$c = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6}$$

$$c = 1 \pm \frac{\sqrt{3}}{3}$$

c) $h(x) = \dfrac{x^2 + 2x - 1}{x^3 - x}$ on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

$$h_{ave} = \frac{1}{\frac{1}{2} - \left(-\frac{1}{2}\right)}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x^2 + 2x - 1}{x(x-1)(x+1)}\,dx$$

$$\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{A(x^2-1) + B(x^2+x) + C(x^2-x)}{x^3 - x} = \frac{x^2 + 2x - 1}{x^3 - x}$$

$A + B + C = 1 \longrightarrow B + C = 1 - 1 = 0 \longrightarrow B = -C$

$B - C = 2 \longrightarrow -C - C = 2 \longrightarrow C = -1 \longrightarrow B = 1$

$-A = -1 \longrightarrow A = 1$

$$= \frac{1}{1}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{x} + \frac{1}{x-1} + \frac{-1}{x+1}\,dx = \ln|x| + \ln|x-1| - \ln|x+1|\Big|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \ln\left|\frac{x(x-1)}{x+1}\right|\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}} = \ln\left|\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{\frac{3}{2}}\right| - \ln\left|\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{\frac{1}{2}}\right|$$

$$= \ln\left(\frac{1}{3}\right) - \ln(3) = \ln\left(\frac{\frac{1}{3}}{3}\right) = \ln\left(\frac{1}{9}\right)$$

$h(c) = h_{ave} \Rightarrow \dfrac{c^2 + 2c - 1}{c^3 - c} = \ln\left(\frac{1}{9}\right) \longrightarrow$ GROSS! DON'T DO IT!

it won't be on the final ◡