Math 2 March 4, 2008

Final Review

GOOD THINGS TO KNOW...

1. The graphs of all six trigonometric functions and their values at standard angles.
2. The antiderivatives of the following functions:

$x^n, n \neq -1$	$\frac{1}{x}$	e^x
$\cos x$	$\sin x$	$\sec^2 x$
$\sec x \tan x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{1+x^2}$

3. The meaning of position, distance, displacement, velocity, speed, and acceleration and how they relate to one another using derivatives and definite and indefinite integrals.
4. The definition of the definite integral as a limit of a Riemann sum.
5. Properties of the definite integral.
6. The statement, including hypotheses, of both parts of the Fundamental Theorem of Calculus.
7. The Substitution Rule for Definite Integrals.
8. The calculus definition of area between curves.
9. The calculus definition of the volume of a solid.
10. The definition of force and work, when other quantities are constant and when they aren't.
11. The statement of the Mean Value Theorem for Integrals.
12. The formula for integration by parts.
13. The method used to evaluate integrals of the form:

$$\int \sin^m x \cos^{2k+1} x \, dx \qquad \int \sin^{2k+1} x \cos^n x \, dx$$

$$\int \tan^m x \sec^{2k} x \, dx \qquad \int \tan^{2k+1} x \sec^n x \, dx$$

14. The helpful trig substitution for all three "square root" cases, and which identities are helpful.

SOME PROBLEMS FOR PRACTICE

1. Consider the region bounded by $y = 2x - 1$, $y = -1$, and $x = 2$.
   (a) Find the area of the region on three ways. You should get the same answer all three ways.
      i. Use geometric formulas.
      ii. Express it as the limit of a Riemann Sum, and finding the limit.
      iii. Change the Riemann Sum above into an integral and evaluate.
   (b) Find the volume of the solid created when this region is rotated about the line $y = -1$ in three ways. Again you should get the same answer each time.
      i. Use one or more geometric formulas.
      ii. Use an integral arrived at through slicing.
      iii. Use an integral arrived at through cylindrical shells.
   (c) Find the volume of the solid created when this region is rotated about the y-axis.
      i. Use one or more geometric formulas.
      ii. Use an integral arrived at through slicing.
      iii. Use an integral arrived at through cylindrical shells.
2. Find the derivative of the following functions.
   (a) $f(x) = \int_a^b g(t)\, dt$
   (b) $F(x) = \int_x^1 \sqrt{t + \sin t}\, dt$
   (c) $G(x) = \int_0^x \frac{t^2}{1+t^3}\, dt$
   (d) $y = \int_{\sqrt{x}}^x \frac{e^t}{t}\, dt$
3. A particle moves along a line with velocity function $v(t) = t^2 - t$, where v is measured in meters per second. Find
   (a) the displacement of the particle during the time interval $[0, 5]$,
   (b) and the distance traveled by the particle over the same time interval.
4. Evaluate:
   (a) $\int_0^1 \frac{d}{dx}(e^{\arctan x})\, dx$
   (b) $\frac{d}{dx} \int_0^1 e^{\arctan x}\, dx$
   (c) $\frac{d}{dx} \int_0^x e^{\arctan t}\, dt$
5. Find the area between the given curves.
   (a) $y = x^2$ and $x = y^2$
   (b) $y = \tan x$, $y = 2\sin x$, and $-\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$
   (c) $y = |x|$ and $y = \frac{2}{x^2+1}$

6. Use calculus to find the area of the triangle with vertices (0.0), $(2, 1)$ and $(-1, 6)$.

7. Let R be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Calculate the following:

   (a) The area of R.

   (b) The volume obtained by rotating R about the x-axis.

   (c) The volume obtained by rotating R about the y-axis.

8. Find the volume of the solid formed when the given region is rotated about the given axis.

   (a) $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 3$; about $y = -1$.

   (b) $y = -x^2 + 6x - 8$ and $y = 0$; about the x-axis.

   (c) $x = (y - 3)^2$ and $x = 4$; about $y = 1$.

   (d) $y = \ln x$, $y = 0$, and $x = 2$; about the y-axis.

   (e) $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$, and $x = 2$; about $x = 2$.

9. Find the volume of the solid described below.

   (a) The base of the solid is a circular disk with radius 3. The parallel cross-sections perpendicular to the base are isosceles right triangles with the hypotenuse lying along the base.

   (b) The base of the solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. The cross sections perpendicular to the x-axis are squares with one side lying along the base.

10. Use calculus to verify the following geometric formulas:

    (a) The area of a circle with radius r: $A = \pi r^2$.

    (b) The volume of a sphere of radius r: $V = \frac{4}{3}\pi r^3$.

    (c) The volume of a cone with radius r and height h: $V = \frac{1}{3}\pi r^2 h$.

11. Evaluate the following integrals:

    (a) $\int_1^4 \frac{dt}{(2x+1)^3}$

    (b) $\int_0^1 \frac{\sqrt{\arctan x}}{x^2+1}\, dx$

    (c) $\int \frac{1}{y^2-4y-12}\, dy$

    (d) $\int \frac{\sec^6 \theta}{\tan^2 \theta}\, d\theta$

    (e) $\int te^{\sqrt{t}}\, dt$

    (f) $\int \frac{1-\tan\theta}{1+\tan\theta}\, d\theta$

    (g) $\int_0^{\frac{\pi}{4}} \tan^5 \theta \sec^3 \theta\, d\theta$

    (h) $\int x^2 \sin x\, dx$

12. A spring has a natural length of 20 cm. Compare the work, W_1, done in stretching the spring from 20 cm to 30 cm with the work, W_2, done in stretching it from 30 cm to 40 cm. How are W_1 and W_2 related?

13. An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the fact that the density of water is 1000 kg/m^3.)

14. Find the average value of the given function on the given interval. Also fin d the value, c where the function attains its average value.

    (a) $f(x) = \sqrt{x}$ on $[0, 4]$.

    (b) $g(x) = 2 + 6x - 3x^2$ on $[0, 2]$.

    (c) $h(x) = \frac{x^2+2x-1}{x^3-x}$ on $\left[-\frac{1}{2}, \frac{1}{2}\right]$.