1. TRUE OR FALSE! Mark each statement TRUE, only if it is always true, and FALSE if it is not always true. There will be no partial credit for these problems.

(a) [3 points] If f is continuous on $[a,b]$, then

$$\sqrt{\int_a^b f(x)\,dx} = \int_a^b \sqrt{f(x)}\,dx$$

FALSE: try $f(x)=1$

(b) [3 points] If f and g are continuous on $[a,b]$, then

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

TRUE

(c) [3 points] $\int_{-1}^{3} 2x - x^2\,dx$ represents the area between the curve

$$y = 2x - x^2$$

the x-axis, $x=-1$ and $x=3$.

FALSE - represents net-area

(d) [3 points] If f is continuous on $[a,b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x).$$

FALSE: $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = 0$

2. Find the derivative of $g(x)$ in two ways:

$$g(x) = \int_x^5 3t^3 + 2t\, dt$$

(a) [6 points] Use the Part 1 of the Fundamental Theorem of Calculus to compute $g'(x)$.

$$g(x) = \int_x^5 3t^3+2t\ dt = -\int_5^x 3t^3+2t\,dt$$

$$g'(x) \underset{\uparrow}{=} -(3x^3+2x) = -3x^3-2x$$

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(b) [6 points] Use the second part of the Fundamental Theorem to compute the definite integral, $g(x) = \int_x^5 3t^3 + 2t\, dt$, then take its derivative.

$$g(x) = \int_x^5 3t^3+2t\ dt = \left[\frac{3t^4}{4}+t^2\right]_x^5$$

$$= \left(\frac{3(5)^4}{4}+5^2\right) - \left(\frac{3x^4}{4}+x^2\right)$$

$$g'(x) = 0+0-4\cdot\frac{3x^3}{4}-2x = -3x^3-2x \checkmark$$

3. Calculate the exact value of the following integral in three different ways.

$$\int_1^5 3 + t\,dt$$

All three ways should give you the same answer.

(a) [8 points] Use the definition of a definite integral as the limit of a Riemann Sum.

$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n}$

$x_i = a + i\Delta x$

$= 1 + \frac{4i}{n}$

$f(x_i) = 3 + (1 + \frac{4i}{n})$

$= 4 + \frac{4i}{n}$

$\int_1^5 3+t\,dt$

$= \lim_{n\to\infty} \sum_{i=1}^{n} (4 + \frac{4i}{n})(\frac{4}{n})$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{16}{n} + \frac{16i}{n^2}$

$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{16}{n} + \sum_{i=1}^{n} \frac{16i}{n^2}\right)$

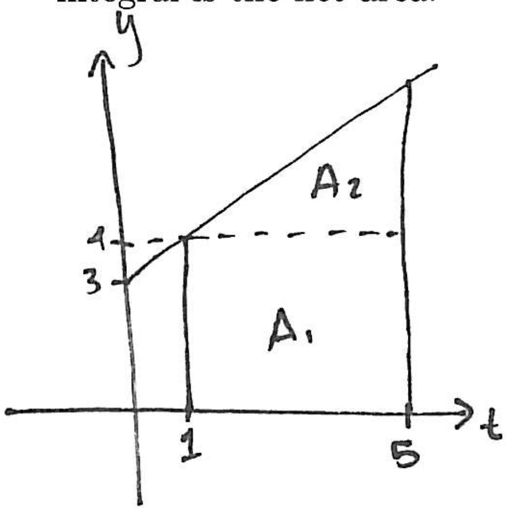
$= \lim_{n\to\infty} \left(\frac{16}{n}\sum_{i=1}^{n} 1 + \frac{16}{n^2}\sum_{i=1}^{n} i\right)$

$= \lim_{n\to\infty} \left(\frac{16}{n}(n) + \frac{16}{n^2}\left(\frac{n(n+1)}{2}\right)\right)$

$= \lim_{n\to\infty} \left(16 + \frac{16}{2} \cdot \cancel{\frac{n+1}{n}}^{\,1}\right)$

$= 16 + 8 = 24$

(b) [8 points] Use basic geometric formulas, and the fact that the definite integral is the net area.



$A_1 = l \cdot h = (5-1)(3+1)$
$= 4 \cdot 4 = 16$

$A_2 = \frac{1}{2} b \cdot h = \frac{1}{2}(5-1)[(5+3)-4]$
$= \frac{1}{2} 4 \cdot 4 = 8$

$A = 16 + 8 = 24$

(c) [8 points] Use the second part of the Fundamental Theorem of Calculus.

$$A = \int_1^5 3+t\,dt = \left[3t + \frac{t^2}{2}\right]_1^5$$
$$= \left(3(5) + \frac{5^2}{2}\right) - \left(3(1) + \frac{1^2}{2}\right)$$
$$= 15 + \frac{25}{2} - 3 - \frac{1}{2}$$
$$= 12 + \frac{24}{2} = 12 + 12 = 24.$$

4. Compute the following integrals. Simplify your answers. (This means evaluate trigonometric, logarithmic, and exponential functions within reason. You do not need to multiply out some large number to another large power, or magically know what $e^{4.5}$ is without a calculator. If you have any questions about how your final answer should look, **please ask.**)

   (a) [10 points] $\int (8x^3 + 3x^2)\,dx$

$$= \frac{8x^4}{4} + \frac{3x^3}{3} + C = 2x^4 + x^3 + C$$

   (b) [10 points] $\int_0^1 \sin(3\pi t)\,dt$

Let $u = 3\pi t$

$du = 3\pi\,dt$

$\frac{du}{3\pi} = dt$

$t = 0 \to u = 0$

$t = 1 \to u = 3\pi$

$$= \int_0^{3\pi} \sin(u)\,\frac{du}{3\pi}$$

$$= \frac{1}{3\pi}\int_0^{3\pi} \sin u\,du$$

$$= \frac{1}{3\pi}\Big[-\cos u\Big]_0^{3\pi}$$

$$= \frac{1}{3\pi}\Big(-\cos(3\pi) - (-\cos(0))\Big)$$

$$= \frac{1}{3\pi}\Big(-(-1) - (-(1))\Big)$$

$$= \frac{1}{3\pi}(1+1) = \frac{2}{3\pi}$$

(c) [10 points] $\int_1^3 \frac{dy}{5y}$

$$\int_1^3 \frac{dy}{5y} = \frac{1}{5}\int_1^3 \frac{1}{y}\,dy$$

$$= \frac{1}{5}\left[\ln|y|\right]_1^3$$

$$= \frac{1}{5}\left(\ln|3| - \ln|1|\right)$$

$$= \frac{1}{5}\left(\ln 3 - \cancel{\ln 1}^{\,0}\right)$$

$$= \frac{\ln 3}{5}$$

5. Suppose a particle moves along a straight line with velocity $v(t)$, where v is in meters per second and time, t, is in seconds. What are the meanings and units of the following quantities? (Your answers should be in terms of the particle's motion. In other words, don't just say $\int_{t_1}^{t_2} v(t)\,dt$ is the integral of the velocity function.)

   (a) [5 points] $|v(t)|$

   the speed of the particle at time t in meters per second ($^m/_s$)

   (b) [5 points] $\int_{t_1}^{t_2} v(t)\,dt$

   the displacement, or net distance, of the particle during the time between t_1 and t_2 in meters (m)

   (c) [5 points] $\int_{t_1}^{t_2} |v(t)|\,dt$

   the total distance traveled on the time interval $[t_1, t_2]$ in meters (m).

   (d) [5 points] $v'(t)$

   the acceleration of the particle at time t in meters per second^2 ($^m/_{s^2}$)

6. [2 points] What is the coolest thing you've learned in Math 2 so far? (Note: Any answer will receive 2 points. Not answering will get you no points.) The FUNdamental Theorem of Calculus of course.

7. BONUS [5 points] You may remember from sometime in your math career that an odd function is defined to be any function such that

$$f(-x) = -f(x)$$

for all values of x. Use this and the substitution rule for definite integrals to show that for an odd function f,

$$\int_{-a}^{a} f(x)\,dx = 0$$

Suppose $f(x)$ is <u>any</u> odd function.

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx$$

$$\int_{-a}^{0} f(x)dx = \int_{a}^{0} f(-u)(-du) = -\int_{a}^{0} f(-u)du$$

$u = -x$

$du = -dx$

$x = -a \rightarrow u = a$

$x = 0 \rightarrow u = 0$

$$= --\int_{0}^{a} f(-u)du$$

$$= \int_{0}^{a} -f(u)du \quad \text{since } f \text{ is odd } f(-u) = -f(u)$$

$$= -\int_{0}^{a} f(u)du$$

$$\int_{-a}^{a} f(x)dx = -\int_{0}^{a} f(u)du + \int_{0}^{a} f(x)dx$$

$$= -\int_{0}^{a} f(t)dt + \int_{0}^{a} f(t)dt$$

$$= 0.$$

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