NAME: $\qquad$

## Math 2 Exam 1

January 31, 2008

Instructions: This is a closed book, closed notes exam. You are not to provide or receive help from any outside source during the exam.

- Print your name clearly in the space provided.
- You may not use a calculator.


## Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own.

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 12 |  |
| 2 | 12 |  |
| 3 | 24 |  |
| 4 | 30 |  |
| 5 | 20 |  |
| 6 | 2 |  |
| 7 | 0 |  |
| Total: | 100 |  |

1. TRUE OR FALSE! Mark each statement TRUE, only if it is always true, and FALSE if it is not always true. There will be no partial credit for these problems.
(a) [3 points] If $f$ is continuous on $[a, b]$, then

$$
\sqrt{\int_{a}^{b} f(x) d x}=\int_{a}^{b} \sqrt{f(x)} d x
$$

(b) [3 points] If $f$ and $g$ are continuous on $[a, b]$, then

$$
\int_{a}^{b} f(x) \pm g(x) d x=\int_{a}^{b} f(x) d x \pm \int_{a}^{b} g(x) d x
$$

(c) [3 points] $\int_{-1}^{3} 2 x-x^{2} d x$ represents the area between the curve

$$
y=2 x-x^{2}
$$

the $x$-axis, $x=-1$ and $x=3$.
(d) [3 points] If $f$ is continuous on $[a, b]$, then

$$
\frac{d}{d x}\left(\int_{a}^{b} f(x) d x\right)=f(x)
$$

2. Find the derivative of $g(x)$ in two ways:

$$
g(x)=\int_{x}^{5} 3 t^{3}+2 t d t
$$

(a) [6 points] Use the Part 1 of the Fundamental Theorem of Calculus to compute $g^{\prime}(x)$.
(b) [6 points] Use the second part of the Fundamental Theorem to compute the definite integral, $g(x)=\int_{x}^{5} 3 t^{3}+2 t d t$, then take its derivative.
3. Calculate the exact value of the following integral in three different ways.

$$
\int_{1}^{5} 3+t d t
$$

All three ways should give you the same answer.
(a) [8 points] Use the definition of a definite integral as the limit of a Riemann Sum.
(b) [8 points] Use basic geometric formulas, and the fact that the definite integral is the net area.
(c) [8 points] Use the second part of the Fundamental Theorem of Calculus.
4. Compute the following integrals. Simplify your answers. (This means evaluate trigonometric, logarithmic, and exponential functions within reason. You do not need to multiply out some large number to another large power, or magically know what $e^{4.5}$ is without a calculator. If you have any questions about how your final answer should look, please ask.)
(a) [10 points] $\int\left(8 x^{3}+3 x^{2}\right) d x$
(b) [10 points] $\int_{0}^{1} \sin (3 \pi t) d t$
(c) $[10$ points $] \int_{1}^{3} \frac{d y}{5 y}$
5. Suppose a particle moves along a straight line with velocity $v(t)$, where $v$ is in meters per second and time, $t$, is in seconds. What are the meanings and units of the following quantities? (Your answers should be in terms of the particle's motion. In other words, don't just say $\int_{t_{1}}^{t_{2}} v(t) d t$ is the integral of the velocity function.)
(a) $[5$ points] $|v(t)|$
(b) [5 points] $\int_{t_{1}}^{t_{2}} v(t) d t$
(c) [5 points] $\int_{t_{1}}^{t_{2}}|v(t)| d t$
(d) [5 points] $v^{\prime}(t)$
6. [2 points] What is the coolest thing you've learned in Math 2 so far? (Note: Any answer will receive 2 points. Not answering will get you no points.)
7. BONUS [5 points] You may remember from sometime in your math career that an odd function is defined to be any function such that

$$
f(-x)=-f(x)
$$

for all values of $x$. Use this and the substitution rule for definite integrals to show that for an odd function $f$,

$$
\int_{-a}^{a} f(x) d x=0
$$

