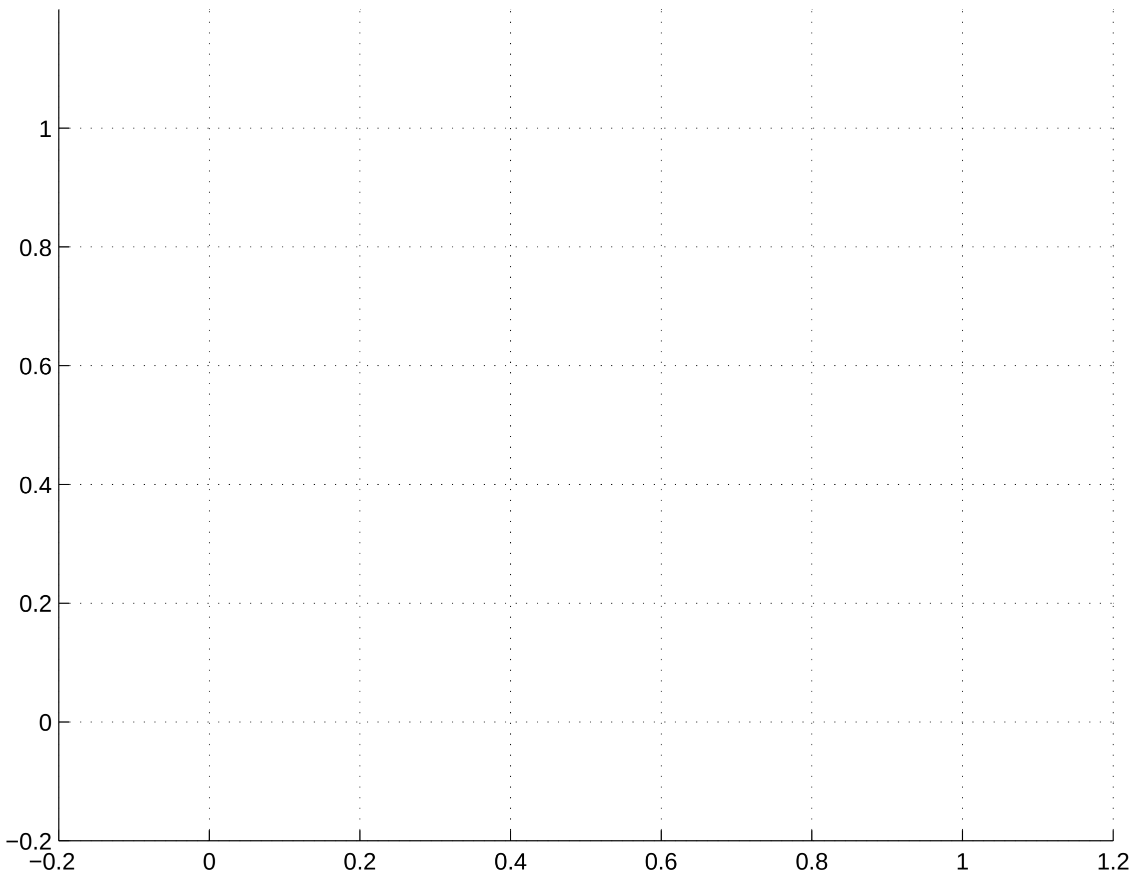
Name and Section:________________________

Instructor's Name:________________________

Group Work 2

1. Draw a graph of the function $f(x) = x^2$. Make it nice and big so it will be easy to draw on.



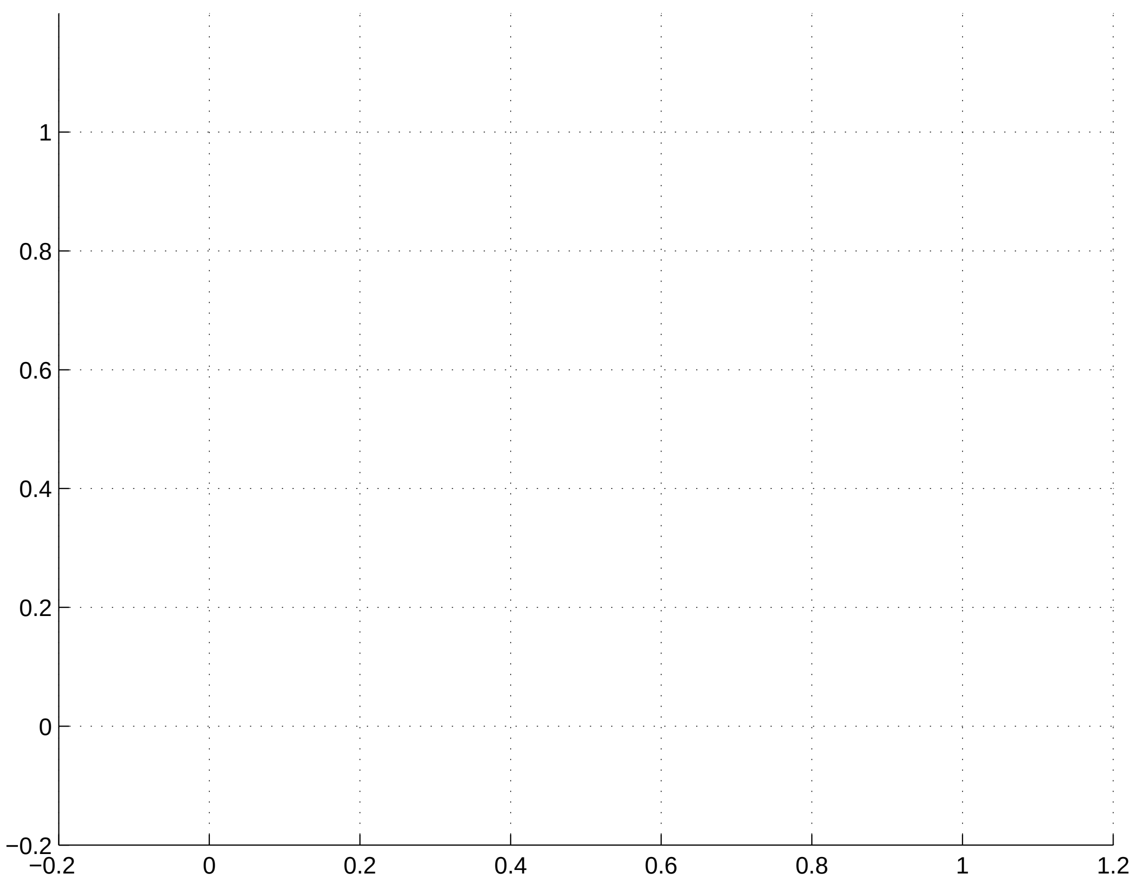
2. We are going to compute the area under $f(x)$ between $x = 0$ and $x = 1$. Lightly shade this region to mark the area we want to compute.

3. We will begin by approximating the area using rectangles. Divide the interval between 0 and 1 into 3 equal length intervals. Using the left endpoints of each of the 3 intervals draw three rectangles that approximate the area under the curve.

4. What is the area of the approximating region, that is, the total area of the three rectangles?

5. This is not the best approximation we could do. How can we make it better?

6. Repeat question 3 using 10 intervals instead of 3.



7. What is the area of the new approximating region?

8. If we do this with more and more rectangles we will get approximations that are closer and closer to the actual answer. In order to find the exact area we must compute a limit. The first step is to write an expression for the area of the approximating region if n rectangles are used. Do this now by spotting the pattern in problem 4 and 7.

9. This is not a fact that you need to have memorized, but we know that $1^2+2^2+3^2+\cdots+(n-1)^2+n^2 = \frac{n(n+1)(2n+1)}{6}$. Use this fact to simplify your expression in the previous question.

10. Compute the limit as $n \to \infty$ of the above expression. This will take some algebraic manipulation.

11. In words, what is this limit saying about the successive approximations to the area under the parabola?