Final Exam Study Guide

March 8, 2007

Disclaimer: This is intended to be a study guide for important concepts that will be covered on the exam. It is not a comprehensive list of all types of problems on the exam. ALL webwork and book problems assigned, as well as quiz problems are fair game for the exam.

1. $f''(x) = \sin(x)$.
   (a) What is $f(x)$? (HINT: There are two integration constants)
   (b) What is $f(x)$ if $f'(0) = 1$ and $f(2) = 4$?
2. A ball is thrown upward with an initial velocity of $1m/s$. Assuming the acceleration due to gravity is $-10m/s^2$, how high does the ball go up before turning around and coming back down?
3. Compute the right endpoint Riemann sum for the function $f(x) = x^2$ between $x = 1$ and $x = 3$ using n rectangles.
4. Using the above, compute the area underneath $f(x) = x^2$ between $x = 1$ and $x = 3$. (HINT: You can check your answer by integrating.)
5. Compute the following integrals. You will need to either integrate by parts or make a substitution.
   (a) $\int x\cos(x^2)dx$
   (b) $\int 5x\cos(x)dx$
   (c) $\int x^4\ln(x)dx$
   (d) $\int \frac{\cos(x)}{1+(\sin(x))^2}dx$
   (e) $\int \sin(x)e^{3x}dx$
   (f) $\int \ln(x)dx$
   (g) $\int x^2\ln(x^3)dx$
6. Compute the following definite integrals.

(a) $\int_0^{\pi}(4+x)\sin(x)dx$

(b) $\int_1^9 \frac{(\pi)^2 x^9}{8+x^{10}}dx$

(c) $\int_0^{\frac{\pi}{4}}(\cos(x)-\sin(x))dx$

(d) $\int_{-\pi}^{\pi}(x+x^2)e^x dx$

(e) $\int_1^2 x\ln(x)dx$

7. Compute the following indefinite integrals by using parts and substitution.
   (a) $\int 5\arctan(x)dx$
   (b) $\int x^5\cos(x^3)dx$

8. Consider the region bounded by the curves $y = 2x^2$, $y = 4$, and $x = 0$.
   (a) Find the area of the above region.
   (b) Find the volume of the solid obtained by rotating the above region around the x-axis.
   (c) Find the volume of the solid obtained by rotating the above region around the y-axis.

9. Consider the region bounded by the curves $y = -x^2 + 4$ and $y = x^2$.
   (a) Find the area of the above region.
   (b) Set up (but do not solve) an integral which corresponds to the volume of the solid obtained by revolving the region around the line $x = 10$.
   (c) Set up (but do not solve) an integral which corresponds to the volume obtained by revolving the region around the line $y = 10$.

10. Consider the region bounded by the curves $y = \sin(x)$, $y = \cos(x)$, $x = \pi/4$, and $x = \pi/2$.
    (a) Find the area of the region.
    (b) Set up (but do not solve) an integral which corresponds to the volume of the solid obtained by revolving the region around the line $x = 0$.
    (c) Set up (but do not solve) an integral which corresponds to the volume of the solid obtained by revolving the region around the line $y = 0$.

11. Consider the following improper integrals. For each deduce wether it converges or diverges. If it converges, compute the value. If it diverges, explain why.
    (a) $\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx$
    (b) $\int_1^{\infty}\frac{1}{x^2}dx$

(c) $\int_1^\infty \frac{1}{x} dx$

(d) $\int_0^2 \frac{5}{x-1} dx$

(e) $\int_2^5 (x-2)^{-.3} dx$

(f) $\int_0^1 \ln(x) dx$

(g) $\int_0^1 \frac{1}{\sqrt{x}} dx$

12. Consider the region bounded by the curves $y = \frac{1}{x}$, $y = 0$, and $x = 0$.

    (a) Find the volume of the solid obtained by rotating this region around the x-axis.

    (b) Find the volume of the solid obtained by rotating this region around the y-axis.