Name and Section: $\qquad$
Instructor's Name: $\qquad$

## Math 2 Midterm 2

February 19, 2007
Instructions: This is a closed book, closed notes exam. You are not allowed to provide or receive help from any outside source during the exam.

- Print your name, section number and instructor in the space provided.
- No calculators are allowed.
- You must show your work to receive full credit.


## Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own.

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 24 |  |
| 2 | 24 |  |
| 3 | 10 |  |
| 4 | 10 |  |
| 5 | 12 |  |
| 6 | 8 |  |
| 7 | 8 |  |
| 8 | 8 |  |
| 9 | 10 |  |
| 10 | 16 |  |
| 11 | 12 |  |
| 12 | 8 |  |
| 13 | 0 |  |
| Total: | 150 |  |

1. Compute the following integrals using the substitution method:
(a) $[8$ points $]$

$$
F(x)=\int 2 x \sin \left(x^{2}\right) d x
$$

(b) $[8$ points $]$

$$
F(x)=\int \frac{5 x^{3}}{3+x^{4}} d x
$$

(c) $[8$ points $]$

$$
\int_{1}^{e} \frac{1}{x} \ln ^{2}(x) d x
$$

2. Compute the following integrals using the integration by parts method:
(a) $[8$ points]

$$
F(x)=\int x e^{x} d x
$$

(b) $[8$ points $]$

$$
\int_{1}^{2} x \ln (x) d x
$$

(c) $[8$ points $]$

$$
F(x)=\int \sin (x) e^{x} d x
$$

3. [10 points] Compute the following indefinite integral:

$$
F(x)=\int 2 x^{3} \cos \left(x^{2}\right) d x
$$

[Hint: Use substitution first and then parts]
4. [10 points] Compute the following indefinite integral:

$$
F(x)=\int \arcsin (x) d x
$$

[Hint: Remember that $\frac{d}{d x} \arcsin (x)=\frac{1}{\sqrt{1-x^{2}}}$ ]
5. [12 points] Compute the following indefinite integral:

$$
F(x)=\int \frac{x^{2}+1}{x} \ln (x) d x
$$

6. [8 points] Evaluate

$$
\int_{-2}^{2} \sqrt{4-x^{2}} d x
$$

by interpreting the integral in terms of area.
[Hint: $x^{2}+y^{2}=4$ is the equation of a circle of radius 2]
7. [8 points] Shade on the graph and compute the area in between the curves $y=e^{x}$ and $y=x^{3}$ in between the lines $x=-1$ and $x=1$.

8. [8 points] Shade on the graph and compute the area in between the curves $y=x^{2}-4$ and $y=4-x^{2}$.

9. [10 points] Shade on the graph and compute the area in between the curves $y=x e^{x}$ and $y=-x^{2}$ in between the lines $x=-\frac{3}{10}$ and $x=\frac{3}{10}$.

10. Determine if each of the following statements is true or false. If it is true, explain why, if not, find a counterexample, that is, a case that proves the statement is incorrect.
(a) [8 points] If $f$ is a continuous function then

$$
\int f^{2}(x) d x=\frac{f^{3}(x)}{3 f^{\prime}(x)}+C
$$

where $C$ is a constant number.
(b) [8 points] If $f$ and $g$ are continuous functions, then

$$
\int f(x) \cdot g(x) d x=\left(\int f(x) d x\right) \cdot\left(\int g(x) d x\right)
$$

11. [12 points] Given the function $f(x)$ whose graph is drawn below

circle the graph corresponding to one of the antiderivatives of $f(x)$ and explain the reason for your choice.

(c)


(d)

12. [8 points] Find an expression for the function:

$$
F(x)=\frac{d}{d x} \int_{1}^{x^{2}} \frac{e^{t}}{2 t} d t
$$

## Extra credit question

13. Find an expression for the function:

$$
F(x)=\frac{d}{d x} \int_{-1}^{x}\left(\int_{0}^{t^{2}} e^{u} d u\right) d t
$$

[Hint: If you have time left, check that all of your integrals are correct.]

