

NAME AND SECTION: \_\_\_\_\_

INSTRUCTOR'S NAME: \_\_\_\_\_

# MATH 2 MIDTERM 1

January 29, 2007

INSTRUCTIONS: This is a closed book, closed notes exam. You are not allowed to provide or receive help from any outside source during the exam.

- *Print* your name, section number and instructor in the space provided.
- No calculators are allowed.
- You must show your work to receive full credit.

HONOR STATEMENT:

I have neither given nor received help on this exam, and all of the answers are my own.

\_\_\_\_\_  
Signature

Question	Points	Score
1	24	
2	30	
3	10	
4	10	
5	8	
6	8	
7	8	
8	8	
9	22	
10	10	
11	12	
12	0	
Total:	150	

1. Find the derivatives of the following functions:

(a) [4 points]

$$f(x) = \ln(x + 7)$$

(b) [6 points]

$$f(x) = \ln(x + 7)e^x$$

(c) [4 points]

$$f(x) = \sqrt{x}$$

(d) [6 points]

$$f(x) = \cos(\sqrt{x})$$

(e) [4 points]

$$f(x) = \arctan(x)$$

2. Find all the antiderivatives of the following functions:

(a) [6 points]

$$f(x) = e^x + \cos(x)$$

(b) [6 points]

$$f(x) = \frac{4}{1+x^2}$$

(c) [6 points]

$$f(x) = x^3 + 2x - 8$$

(d) [6 points]

$$f(x) = 3x^{7/2} - x^3 + \sqrt{x} - 7x^{-9/2}$$

(e) [6 points]

$$f(x) = \left(x + \frac{1}{x}\right)^2$$

3. [10 points] Find an expression for the function

$$F(x) = \int_0^x (t^2 + \sin t) dt$$

that does not involve the definite integral.

4. Suppose the velocity of a car is given by

$$v(t) = (10t + 4) \text{ ft/s}$$

- (a) [4 points] Find the acceleration of the car at any time  $t$ .

(b) [6 points] How far has the car travelled after 10s?

5. [8 points] Evaluate

$$\int_{-1}^5 |x - 2| dx$$

by interpreting the integral in terms of area.

6. [8 points] Suppose  $f''(x) = \cos(x)$ ,  $f(0) = 0$  and  $f'(0) = 1$ . Find  $f(x)$ .

7. [8 points] Approximate the area under  $f(x) = x^3 - x$  between  $x = 1$  and  $x = 3$  using 4 rectangles and left endpoints.

8. [8 points] Approximate the area under  $f(x) = \sin x$  between  $x = 0$  and  $x = \pi$  using 4 rectangles and right endpoints.

9. Let  $f(x) = x$ .

(a) [6 points] Approximate the area under  $f(x)$  between  $x = 0$  and  $x = 1$  using  $n$  rectangles and right endpoints (i.e. find an expression for  $R_n$ ). For this part, write the answer out longhand.

(b) [5 points] Write the expression for  $R_n$  in  $\Sigma$  notation.

(c) [6 points] Using the fact that  $1 + 2 + \cdots + (n - 1) + n = \frac{n(n+1)}{2}$  find a simpler expression for  $R_n$ .



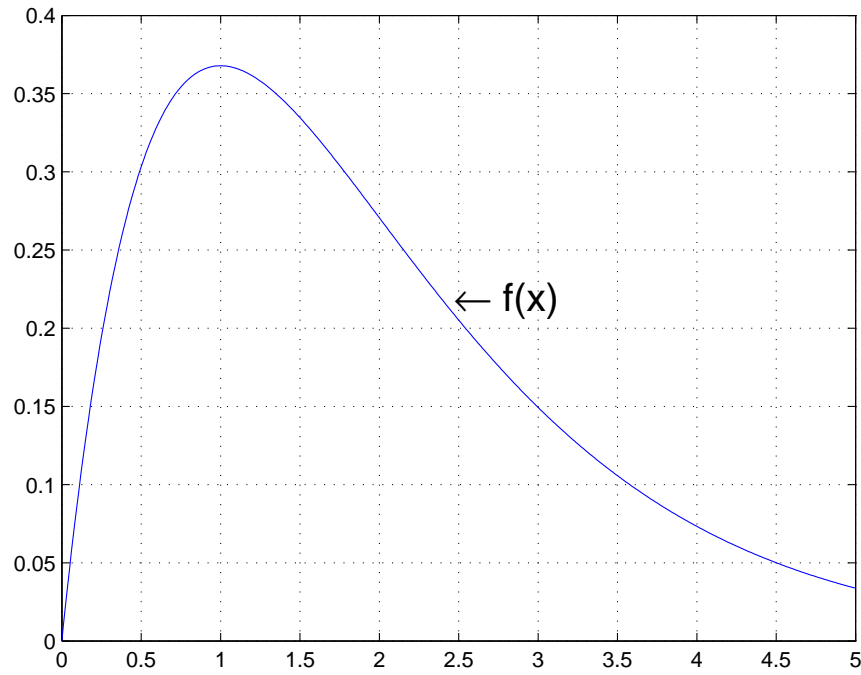
- (d) [5 points] Using the answers to the above parts, find the area under  $f(x) = x$  between  $x = 0$  and  $x = 1$ .

10. [10 points] Find an expression for

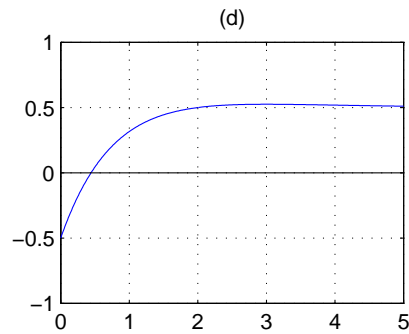
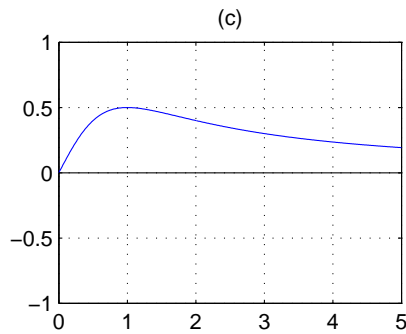
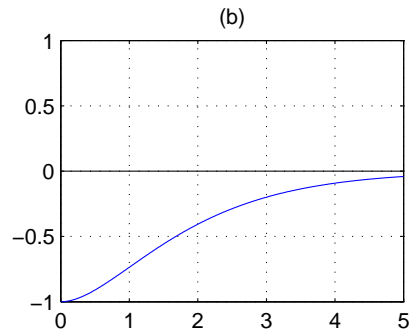
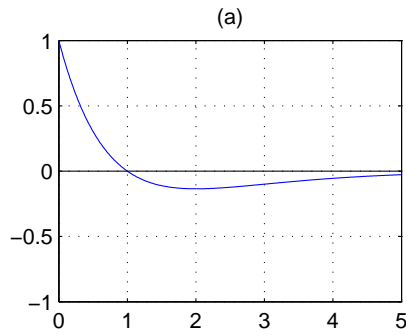
$$\frac{d}{dx} \int_1^{x^2+2x} \frac{1}{2\sqrt{t^3+1}} dt$$

that does not involve the definite integral.

11. [12 points] Given the function  $f(x)$  whose graph is drawn below

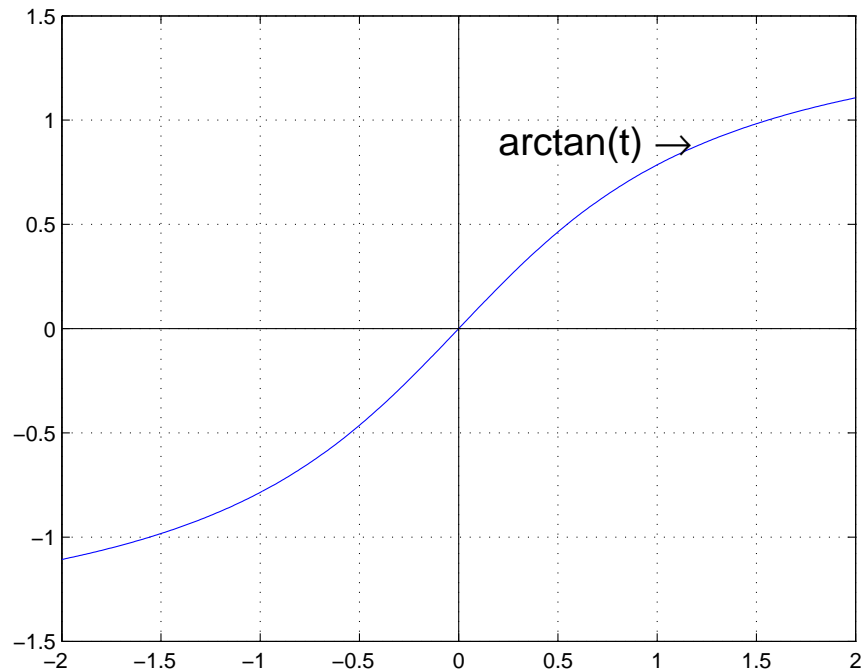


circle the graph corresponding to one of the antiderivatives of  $f(x)$



**Extra credit questions**

12. Consider the function  $f(t) = \arctan(t)$  graphed below



- (a) Shade on the figure the region corresponding to the following definite integral

$$g(x) = \int_{-1}^x \arctan(t) dt$$

- (b) Compute the exact value  $g(1)$  justifying all your steps