Name and Section: _____

Instructor's Name: _____

Math 2 Final

March 11, 2007

Instructions: This is a closed book, closed notes exam. You are not allowed to provide or receive help from any outside source during the exam.

- *Print* your name, section number and instructor in the space provided.
- No calculators are allowed.
- You must show your work to receive full credit.

Question	Points	Score
1	24	
2	24	
3	44	
4	16	
5	30	
6	20	
7	40	
8	20	
9	16	
10	16	
11	0	
Total:	250	

Honor Statement:

I have neither given nor received help on this exam, and all of the answers are my own.

_____

Signature

1. Solve these integrals by substitution:

(a) [6 points]

$$F(x) = \int xe^{3x^2} dx$$

(b) [6 points]

$$F(x) = \int \frac{1}{x^2} e^{1/x} dx$$

(c) [6 points]

$$\int_{-\pi/2}^{\pi/2} \frac{\arctan(x)}{1+x^2} dx$$

(d) [6 points]

$$\int_2^3 x(2-x)^6 dx$$

2. Solve these integrals using integration by parts:

(a) [6 points]

$$F(x) = \int x \cos(2x) dx$$

(b) [6 points]

$$F(x) = \int \sin^2(x) dx$$

(c) [6 points]

$$\int_0^3 x e^{-x} dx$$

(d) [6 points]

$$\int_1^e x^2 \ln(x) dx$$

3. Compute the following integrals:

(a) [10 points]

$$\int_2^2 \left(x^2 + 3 + \frac{2}{x^3}\right)^3 dx$$

(b) [10 points]

$$\int_e^{e^3} \frac{\ln(\ln(x))}{x \ln(x)} dx$$

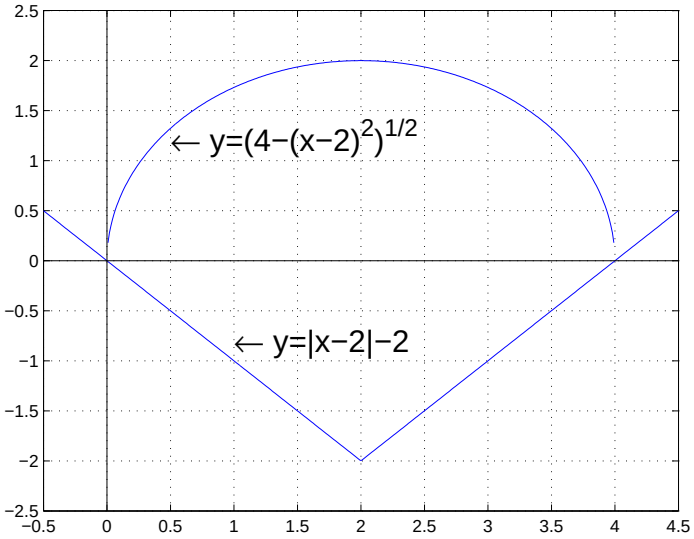
(c) [12 points]

$$F(x) = \int \frac{(1+x)^2}{1+x^2} dx$$

(d) [12 points]

$$F(x) = \int \left(\frac{1}{\sqrt{1+x^2}} + x \right)^2 dx$$

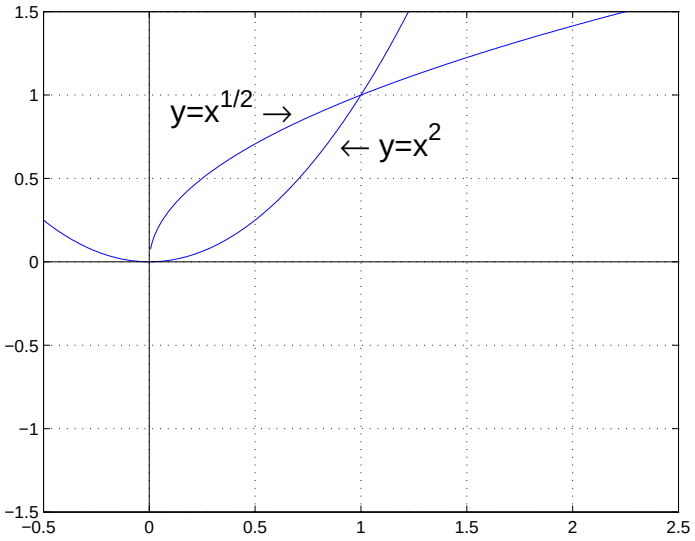
4. [16 points] Let $f(x) = \sqrt{4-(x-2)^2}$ and $g(x) = |x-2| - 2$. Matt's guitar pick can be visualized as the region enclosed in between the curves $y = f(x)$ and $y = g(x)$, as shown in figure.



Compute the definite integral:

$$\int_0^4 (f(x) - g(x))dx$$

5. Consider the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$.



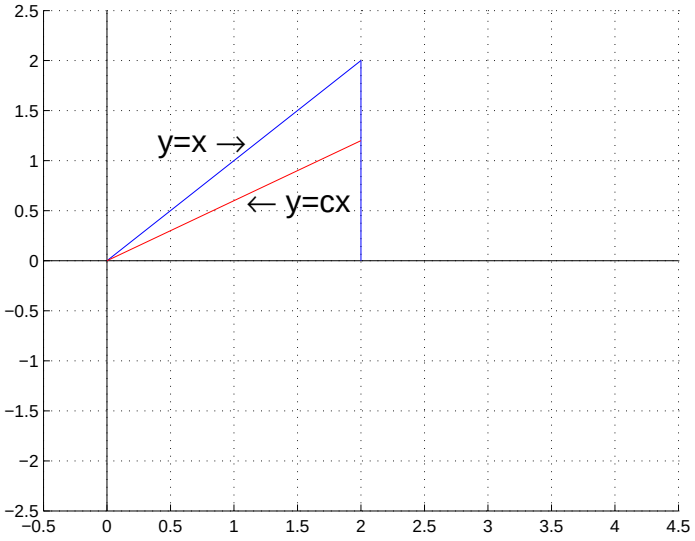
[Hint: A sketch may help you set up the appropriate integrals.]

(a) [8 points] Find the area of the above region.

(b) [10 points] Find the volume of the solid obtained by rotating the above region around the x-axis.

(c) [12 points] Find the volume of the solid obtained by rotating the above region around the line $x = 1$.

6. Let S be the solid obtained by rotating the region in between the curve $y = x$, the x-axis and the line $x = 2$, around the x-axis, that is, a cone of base radius 2 and height 2.



(a) [8 points] Compute the volume of the solid S.

(b) [12 points] Consider the line $y = cx$, for c a constant such that $0 < c < 1$. For which value of the constant c the line splits the above region in two regions such that the corresponding solids, obtained by rotating these regions around the x-axis, have the same volume?

7. Consider the following improper integrals. For each, determine if it converges or diverges. If it converges, then compute its value. If it diverges, then explain why.

(a) [8 points]

$$\int_{10}^{\infty} \frac{1}{x^3} dx$$

(b) [8 points]

$$\int_{-\pi}^{\infty} \sin(x)dx$$

(c) [8 points]

$$\int_{0}^{4} \ln(x)dx$$

(d) [8 points]

$$\int_{0}^{1} x^{-\frac{1}{4}}dx$$

(e) [8 points]

$$\int_{-2}^{2} \frac{1}{x}dx$$

8. A ball is dropped from the top of a $500m$ tall building. Assume that the acceleration due to gravity is $-10m/s^2$.

   (a) [10 points] At what time does the ball to hit the ground?

   (b) [10 points] What is the velocity of the ball when it hits the ground?

9. Determine if each of the following statements is true or false. If it is true, explain why, if not, find a counterexample, that is, a case that proves the statement is incorrect.

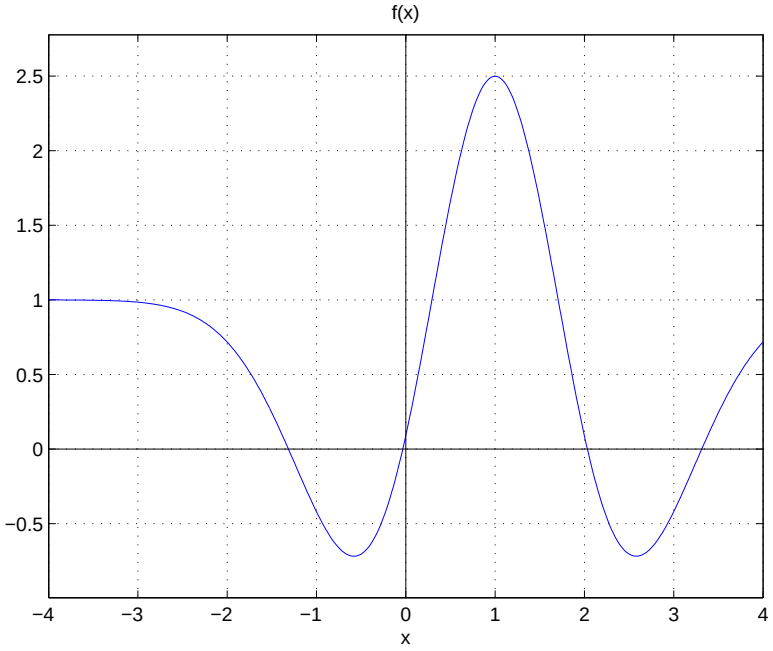
   (a) [8 points] If f and g are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then

$$\int_a^b f(x)\ dx \geq \int_a^b g(x)\ dx$$

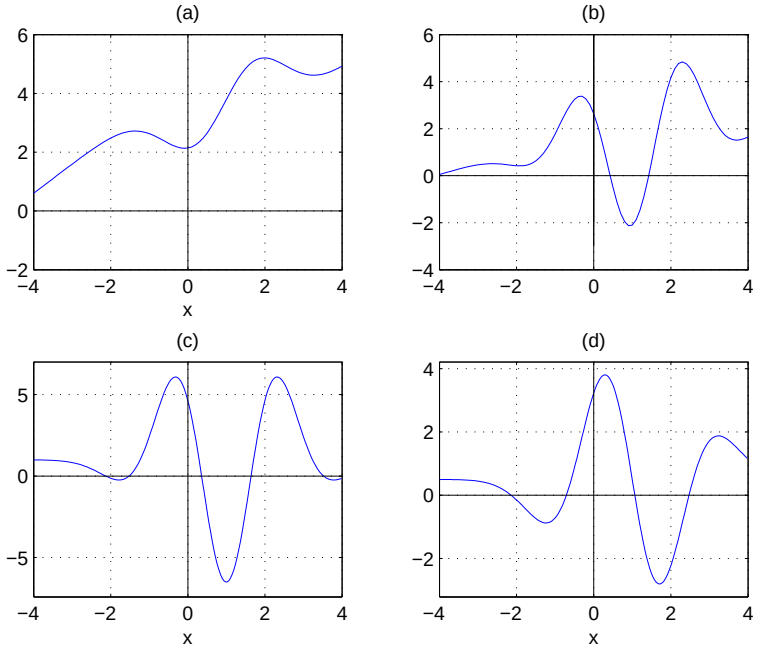
   (b) [8 points] If f and g are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then

$$\int_a^b |f(x)|\ dx \geq \int_a^b |g(x)|\ dx$$

10. [16 points] Given the function $f(x)$ whose graph is drawn below



circle the graph corresponding to one of the antiderivatives of $f(x)$ and explain the reason for your choice.



Extra credit question

11. Given that:

$$\int_0^x f(t)dt = (f(x))^2$$

find an expression for $f(x)$.