Math 2 Winter 2006

Calculus with Algebra and Trigonometry

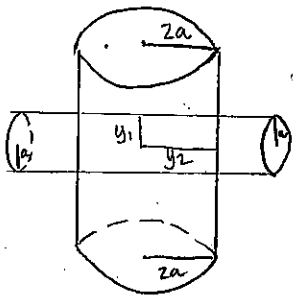
Midterm 2 – Extra credit problem due Friday, February 17 by 4:00pm. You may place it inside of the envelope on your instructor's door if she is not in her office.

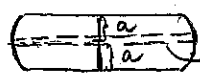
Your Name (Please Print): Solution

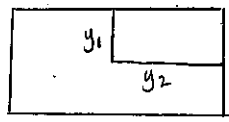
Section (Please Circle): Andersen Henrich

You may use your textbook as well as your notes from this course (but no other sources). The Honor Principle requires that you neither give nor receive any aid on this exam.

(2pt) The axes of two right circular cylinders of radius a and $2a$ intersect at right angles. Describe the cross sections and set up an integral that can be used to find the volume bounded by the cylinders.



solid of intersection  slices like this give you rectangle cross sections.



$y_1 = \sqrt{a^2 - x^2}$ (because comes from circle $x^2 + y^2 = a^2$)

$y_2 = \sqrt{(2a)^2 - x^2} = \sqrt{4a^2 - x^2}$ (because comes from circle $x^2 + y^2 = (2a)^2$)

cross sections are rectangles

So area of rectangle

$$A(x) = (2y_1)(2y_2) = 4y_1y_2 = 4\sqrt{a^2 - x^2}\sqrt{4a^2 - x^2}$$
$$= 4\left(a^4 - 4a^2x^2 - a^2x^2 + x^4\right)^{1/2}$$
$$= 4\left(a^2 - 5a^2x^2 + x^4\right).$$

$$\text{Volume} = \int_{-a}^{a} 4\left(a^2 - 5a^2x^2 + x^4\right)^{1/2} dx$$
$$= 2\int_{0}^{a} 4\left(a^2 - 5a^2x^2 + x^4\right)^{1/2} dx$$
$$= 8\int_{0}^{a} \left(a^2 - 5a^2x^2 + x^4\right)^{1/2} dx$$