Trigonometry: Center Stage

(Note to audience: $*$ denotes a fact that you don't need to memorize.)

The cast:

- $\sin(x)$
- $\cos(x)$
- $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- $\sec(x) = \frac{1}{\cos(x)}$
- $\csc(x) = \frac{1}{\sin(x)}$
- $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{sin(x)}$

Sine and Cosine are co-stars in the following useful trig identity...

$$\sin^2(x) + \cos^2(x) = 1$$

Tangent and Secant are co-stars in the following useful trig identity...

$$1 + \tan^2(x) = \sec^2(x)$$

$*$*Cotangent and Cosecant star in the off-off-Broadway identity...*

$$1 + \cot^2(x) = \csc^2(x)$$

Cosine is getting a lot of work in those Half-Angle Identities, too!

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$
$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

Remember that old triangle act SOHCAHTOA? You know, where

$\sin(x)$ was $\frac{opposite}{hypotenuse}$,

$\cos(x)$ was $\frac{adjacent}{hypotenuse}$,

and $\tan(x)$ was $\frac{opposite}{adjacent}$?

(Those were the days!)

And who could forget our old friend **The Unit Circle**?

"Oh, costume mistress! How quickly can our trig functions change?!"

- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- $*\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$ [Prove this!]
- $*\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$ [Prove this!]
- $*\frac{d}{dx}(\cot(x)) = -\csc^2(x)$ [Prove this!]

"And how can we find out how much area they'll take up??"

- $\int \sin(x)\,dx = -\cos(x) + c$
- $\int \cos(x)\,dx = \sin(x) + c$
- $\int \tan(x)\,dx = -\ln|\cos(x)| + c$ [Prove this!]
- $*\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + c$
- $*\int \csc(x)\,dx = \ln|\csc(x) - \cot(x)| + c$
- $\int \cot(x)\,dx = \ln|\sin(x)| + c$ [Prove this!]

"Last but not least, we'd like to thank our wonderful crew!"

$\sin^{-1}(x)$:

- Helped out $\sin(x)$ in the identity
$$\sin^{-1}(\sin(x)) = x = \sin(\sin^{-1}(x));$$
- Has the wonderful properties
$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}},$$
$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c.$$

$\cos^{-1}(x)$:

- Helped out $\cos(x)$ in the identity
$$\cos^{-1}(\cos(x)) = x = \cos(\cos^{-1}(x));$$
- Has the reliable property
$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}.$$

$\tan^{-1}(x)$:

- Helped out $\tan(x)$ in the identity
$$\tan^{-1}(\tan(x)) = x = \tan(\tan^{-1}(x));$$
- Has the useful properties
$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2},$$
$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c.$$

$\sec^{-1}(x)$:

- Helped out $\sec(x)$ in the identity
$$\sec^{-1}(\sec(x)) = x = \sec(\sec^{-1}(x));$$
- Offered us the following services:
$$\frac{d}{dx}(\sec^{-1}(x)) = \frac{1}{x\sqrt{x^2-1}},$$
$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c.$$

[**Questionnaire**: How can we get $\int \sin^{-1}(x)dx$, $\int \tan^{-1}(x)dx$, etc.?]