MATH 2
PROBLEM OF THE WEEK 7
Due Friday, Feburary 21st, 2003 before the quiz.

Please show all your work!

Name:____

The book shows the formula:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

but doesn't prove it. By answering the following questions, you'll prove it.

(1) Show that

$$\sum_{i=1}^{n} (i+1)^3 - i^3 = (n+1)^3 - 1^3.$$

(2) Show that

$$(i+1)^3 - i^3 = 3i^2 + 3i + 1$$

and conclude that

$$\sum_{i=1}^{n} (i+1)^3 - i^3 = \sum_{i=1}^{n} 3i^2 + 3i + 1.$$

(3) Show that

$$\sum_{i=1}^{n} 3i^2 + 3i + 1 = 3\sum_{i=1}^{n} i^2 + \frac{3n(n+1)}{2} + n$$

(4) Solve for $\sum_{i=1}^{n} i^2$ in

$$(n+1)^3 - 1^3 = 3\sum_{i=1}^{n} i^2 + \frac{3n(n+1)}{2} + n$$

(5) Find a common denominator in your previous answer and show the theorem by multiplying out the right hand side of

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

and seeing the two quantities are equal.