## A Different Way to Learn Mathematics

For the past five years, Professor Bogart in the mathematics department has been teaching some of his courses by a "student discovery" method rather than by lectures. The apparent resulting increase in student understanding has been dramatic enough that the National Science Foundation is supporting development of materials for teaching an introductory major-level course (Math 28, Combinatorics) in this way. The foundation is also supporting the testing of these materials at a broad cross section of colleges and universities across the country.

In a discovery course in mathematics, the text consists almost entirely of problems for students to work on. By working through the problems, students are led to discover and prove for themselves the main ideas of the subject. As well, instead of seeing someone else describe the classical examples of the subject, students see these examples through their own eyes. In most mathematics courses, students spend most of their time working on problems. One difference in discovery learning is that most of the problems students work on lead them through the primary intellectual content of the course, while in a lecture course, the primary intellectual content is in the lectures. In a lecture course, the problems are designed to let the students practice using the main ideas or to students to extend these ideas. In a discovery course, there are practice problems as well, but the majority of a student's work is discovering new (to the student) ideas.

Research in how people learn mathematics suggests that no matter how learners encounter the mathematics they learn, they seem to learn it by making their own mental constructions, connecting these constructions to previously constructed ideas, and reworking the previous constructions when they are inconsistent with the new ones. Discovery learning is designed to foster this process.

The role of the instructor in discovery learning in math is to provide appropriate problems for students to work on, to provide hints, to lead classroom discussions of student work, and to read student solutions, especially for problems that reflect the students' mental constructions, providing feedback for students to use in reworking their solutions as necessary. The materials Professor Bogart is developing now are intended to provide an appropriate set of problems for the study of introductory combinatorics.

In the coming winter term Professor Bogart will be teaching Math 28 using the new materials. Students will be encouraged to work together and to provide feedback to each other on written solutions to some of the problems. In practice, this seems to help students deal with potential dead ends by bringing several different approaches to the solution attempts.

The subject of Math 28 is combinatorial mathematics, the mathematics that analyzes the ways in which we may combine objects into arrangements. Examples of such combinations include pairing up people for games of tennis, choosing a sandwich at a sandwich shop, organizing objects for statistical studies, planning efficient

communications networks, determining gene sequences, and developing efficient computer programs.

Math 28 has only Math 8 or 9 as a prerequisite. The department recommends the course to students who have done reasonably well in one of these courses but who want to study a different kind of mathematics. Bogart's hope is that students who choose this option will now not only study a different kind of mathematics, but will learn how to discover mathematics for themselves.