How to win \$1 million.

The Clay Math Institute (http://www.claymath.org/millennium/) is offering \$1 million to any person who is able to prove a certain conjecture about the Riemann zeta function (see p. 78), called the *Riemann hypothesis*. One possible way to do this involves the Farey series defined on p. 96:

> **Theorem.** Given an integer $n \geq 1$, let $F = \{F_0, F_1, F_2, \ldots F_k\}$ be the Farey series of order n. Then, if the sum
> $$S_n = \sum_{j=1}^{k} \left| F_j - \frac{j}{k} \right| = \mathrm{O}(n^r)$$
> for any $r > \frac{1}{2}$, the Riemann hypothesis is true.

Of course, nobody has shown that the sum S_n satisfies this property in general.

Problem S1: Compute the Farey series for $n = 5$, and show that $S_5 \leq \sqrt{5}$.